Graphical Tools for the Analysis of Bi-objective Optimization Algorithms

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Is a multi-objective optimization algorithm better than another?

Analysis of Multi-objective Optimization Algorithms

- Best criterion: dominance criteria among output sets ($\lhd$)
  - ✔ If $A_r \lhd B_r$ for all r runs, then A is better than B
  - ✘ Output sets of high-performing algorithms are often *incomparable* in terms of dominance
- Unary/binary measures:
  - ✔ Experimental analysis like in single-objective optimization
  - ✔ Intuitively describe desirable properties
  - ✘ Bias
  - ✘ Loss of information (Over-simplification)
- Many unary/binary measures:
  - ✔ Less bias
  - ✔ Less information lost
  - ✘ Difficult interpretation
  - ✘ Consensus issues [Mersmann et al., 2010]

In which aspect
is a multi-objective optimization algorithm
better than another?

The Attainment Function [Grunert da Fonseca et al., 2001]

- Extends scalar concepts of location (mean, median) and spread (variance, IQR) to random sets
- Completely characterizes the statistical distribution of the output of multi-objective optimizers in terms of location, spread and mutual dependence [Fonseca et al., 2005]
- Enables statistical testing and inference [Fonseca et al., 2005; Grunert da Fonseca & Fonseca, 2010; Paquete & Stützle, 2006, 2009]
- Theory more advanced than practical applications

The Attainment Function

[Grunert da Fonseca et al., 2001]

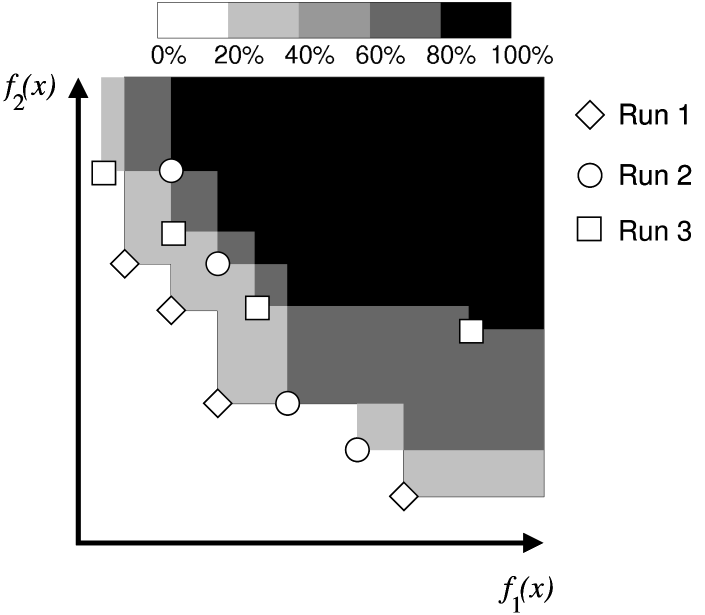
- First-order attainment function:

$$\alpha(\vec{v})\colon \mathbb{R}^d \longrightarrow [0,1]$$

- Probability of a random set *attaining* a particular point $\vec{v}$ in the objective space
- An attainment function can characterize the output of a stochastic multi-objective optimization algorithm
- The real attainment function is unknown but...
- We can *estimate* it:

Empirical attainment function (EAF)

The Empirical Attainment Function



Attainment Surfaces

- k% attainment surface:

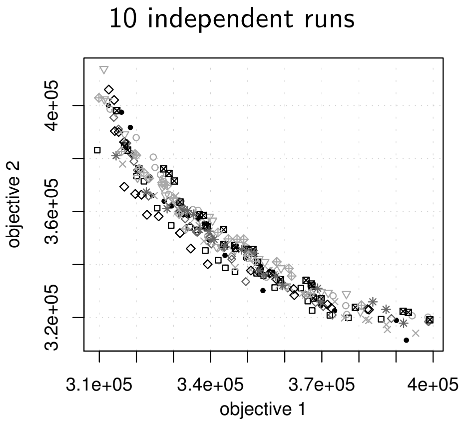
  "Lower boundary of the region in the objective space with a value of the attainment function of at least $k/100$."

- Empirical k% attainment surface:

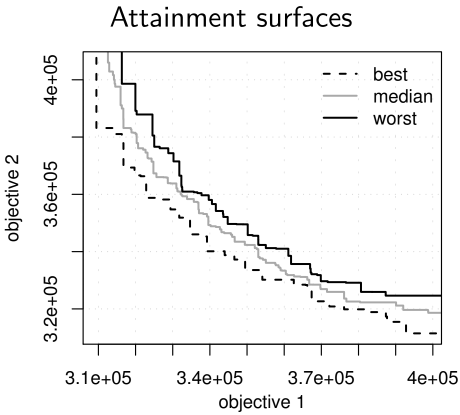
  "The line delimiting the objective space attained by at least k% of the runs of a multi-objective algorithm."

- *Median* attainment surface $=$ 50% attainment surface
- *Worst* attainment surface $=$ 100% attainment surface
- *Best* attainment surface $=$ region attained by at least one run

Attainment Surfaces

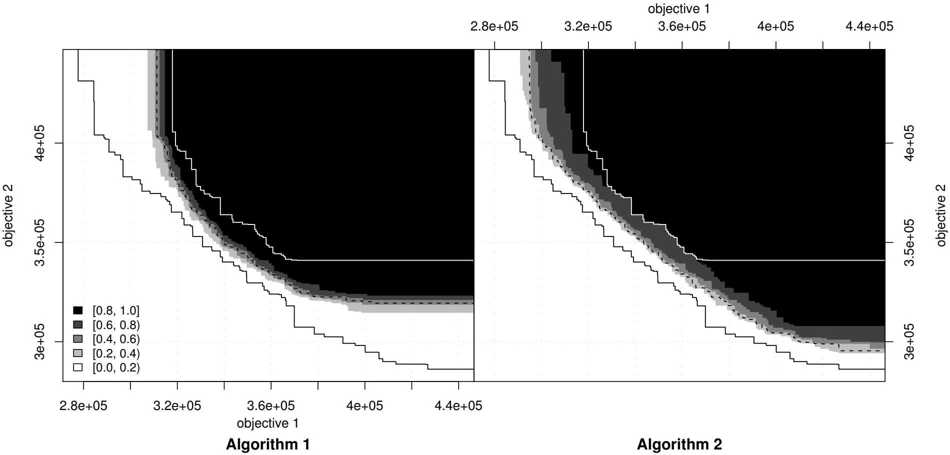


What is the "typical" behaviour?

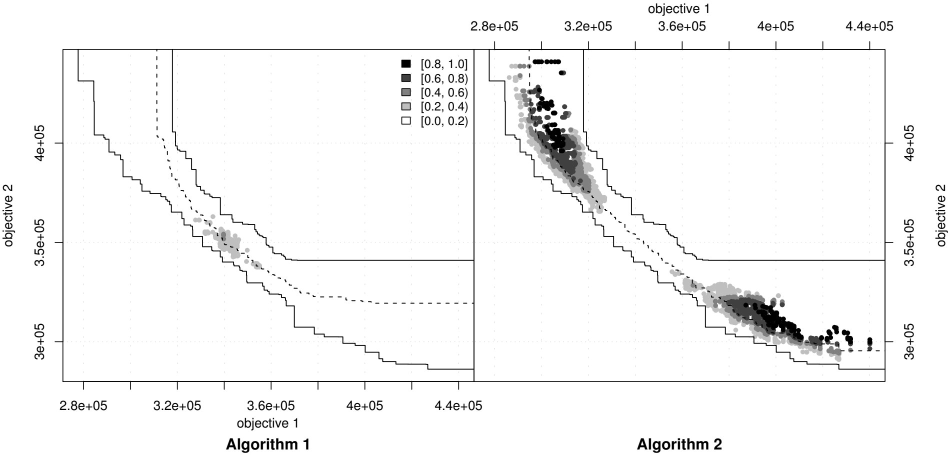


Less clutter, more information!

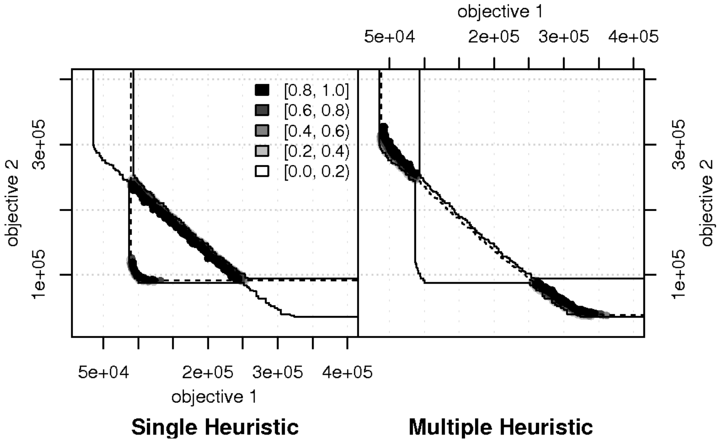
Comparing Two Algorithms: EAFs side-by-side



Comparing Two Algorithms: EAF Differences



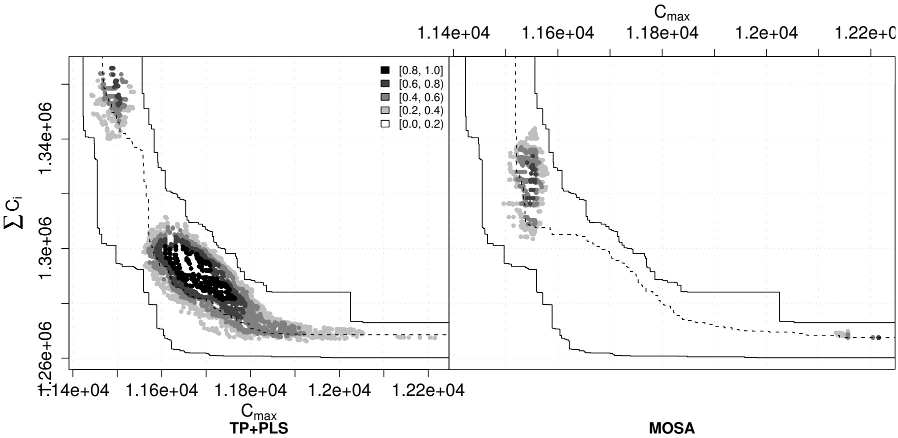
[López-Ibáñez & Stützle, 2010b]



Pareto ACO

EAF Differences: More examples

Permutation Flow-shop, Hybrid TP+PLS against MOSA
[Dubois-Lacoste et al., 2010]



Conclusions

- Not a replacement of dominance or quality measures, but a powerful exploratory data analysis tool
- Related earlier works by Knowles [2005] and Fonseca et al. [2005]
- Ongoing work on both theory and practical applications
- We make available software tools to produce these plots [López-Ibáñez et al., 2010]

http://iridia.ulb.ac.be/~manuel/eaftools

Open Questions for Future Research

1. EAF for more than 2 dimensions
   - No algorithm publicly available (ongoing work)
   - Best way to use the EAF: direct visualization (at most 3D), parallel coordinates, ...
2. How to summarise the results on several instances?
3. Practical applications of higher-order EAFs
4. Theoretical and practical challenges

References I

J. Dubois-Lacoste, M. López-Ibáñez, and T. Stützle. Supplementary material: A Hybrid TPLS+PLS Algorithm for Bi-objective Flow-shop Scheduling Problems. `http://iridia.ulb.ac.be/supp/IridiaSupp2010-001`, 2010.

C. M. Fonseca, V. Grunert da Fonseca, and L. Paquete. Exploring the performance of stochastic multiobjective optimisers with the second-order attainment function. In C. C. Coello, A. H. Aguirre, and E. Zitzler, editors, *Evolutionary Multi-criterion Optimization (EMO 2005)*, volume 3410 of *Lecture Notes in Computer Science*, pages 250–264. Springer, Heidelberg, Germany, 2005.

V. Grunert da Fonseca and C. M. Fonseca. The attainment-function approach to stochastic multiobjective optimizer assessment and comparison. In T. Bartz-Beielstein, M. Chiarandini, L. Paquete, and M. Preuß, editors, *Experimental Methods for the Analysis of Optimization Algorithms*. Springer, 2010.

V. Grunert da Fonseca, C. M. Fonseca, and A. O. Hall. Inferential performance assessment of stochastic optimisers and the attainment function. In E. Zitzler, K. Deb, L. Thiele, C. A. Coello, and D. Corne, editors, *Proceedings of EMO 2001*, volume 1993 of *Lecture Notes in Computer Science*, pages 213–225. Springer, Heidelberg, Germany, 2001.

References II

J. D. Knowles. A summary-attainment-surface plotting method for visualizing the performance of stochastic multiobjective optimizers. In *Proceedings of the 5th International Conference on Intelligent Systems Design and Applications*, pages 552–557, 2005. doi: 10.1109/ISDA.2005.15.

M. López-Ibáñez and T. Stützle. The impact of design choices of multi-objective ant colony optimization algorithms on performance: An experimental study on the biobjective TSP. In *GECCO 2010*. ACM press, New York, NY, 2010a. Accepted.

M. López-Ibáñez and T. Stützle. The impact of design choices of multi-objective ant colony optimization algorithms on performance: An experimental study on the biobjective TSP. `http://iridia.ulb.ac.be/supp/IridiaSupp2010-003/`, 2010b. Supplementary material of López-Ibáñez & Stützle [2010a].

M. López-Ibáñez, L. Paquete, and T. Stützle. Exploratory analysis of stochastic local search algorithms in biobjective optimization. In T. Bartz-Beielstein, M. Chiarandini, L. Paquete, and M. Preuß, editors, *Experimental Methods for the Analysis of Optimization Algorithms*, pages 209–233. Springer, 2010.

O. Mersmann, H. Trautmann, B. Naujoks, and C. Weihs. Benchmarking evolutionary multiobjective optimization algorithms. In H. Ishibuchi et al., editors, *Congress on Evolutionary Computation (CEC)*, page To appear, Piscataway, NJ, 2010. IEEE Press.

References III

L. Paquete and T. Stützle. A study of stochastic local search algorithms for the biobjective QAP with correlated flow matrices. *European Journal of Operational Research*, 169(3):943–959, 2006.

L. Paquete and T. Stützle. Design and analysis of stochastic local search for the multiobjective traveling salesman problem. *Computers & Operations Research*, 36 (9):2619–2631, 2009.