# Optimization Challenges at the European Space Agency


**Dario Izzo**
**European Space Agency**
Dario.Izzo@esa.int

**Manuel López-Ibáñez**
**University of Manchester, UK**
manuel.lopez-ibanez@manchester.ac.uk
http://lopez-ibanez.eu

The University of Manchester
Alliance Manchester Business School


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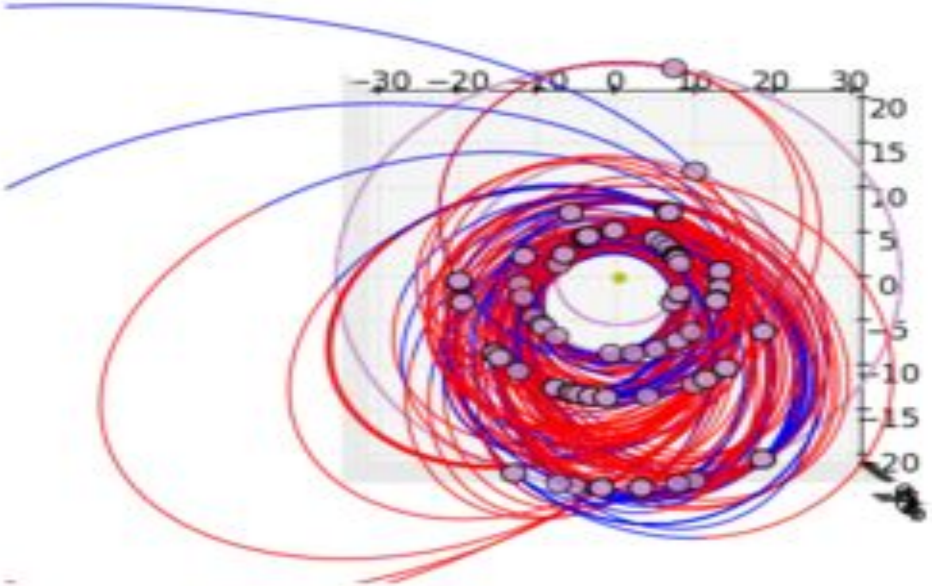
# Instructors

- **Dario Izzo** graduated in Aeronautical Engineering from the University Sapienza of Rome in 1999 and later obtained a second master in “Satellite Platforms” at the University of Cranfield in the UK and a Ph.D. in Mathematical Modelling in 2003, at the University Sapienza of Rome. In 2004, he moved to the European Space Agency (ESA) in the Netherlands as a research fellow in Mission Analysis and, in 2008, he became one of two permanent staff members in the Advanced Concepts Team (ACT). Dr. Izzo is now the head of the ACT managing its interface to the rest of ESA. During the years spent with tha ACT, he has led studies in interplanetary trajectory design and artificial intelligence and he took part in several other innovative researches on diverse fields. He started the Global Trajectory Optimization Competitions events, the ESA’s Summer of Code in Space, and the Kelvins competition platform (https://kelvins.esa.int/). Dr. Izzo has published more than 150 papers in journals, conferences and books. In GECCO 2013, he received the Humies Gold Medal for the work on grand tours of the galilean moons and, the following year, he won the 8th edition of the Global Trajectory Optimization Competition, organized by NASA/JPL, leading a mixed team of ESA/JAXA scientists. His interests range from computer science, open source software development, interplanetary trajectory optimization, biomimetics and artificial intelligence.

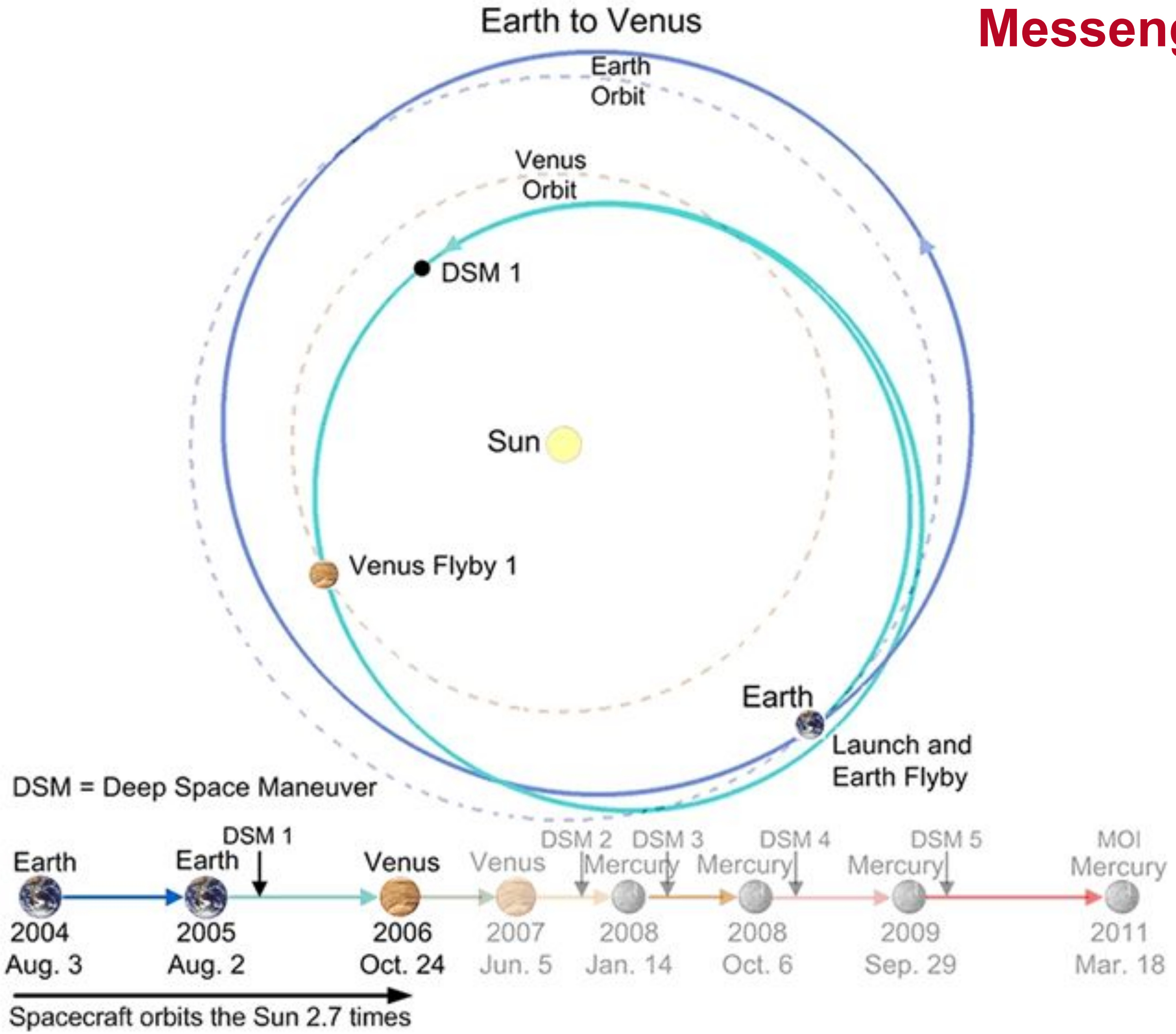
- **Manuel López-Ibáñez** is a Senior Lecturer (Assistant Professor) at the University of Manchester, UK. Between 2020 and 2022, he was a “Beatriz Galindo” Senior Distinguished Researcher at the University of Málaga, Spain. He received the M.S. degree in computer science from the University of Granada, Spain, in 2004, and the Ph.D. degree from Edinburgh Napier University, UK, in 2009. Between 2011 and 2015, he was a Postdoctoral Researcher of the Belgian F.R.S.-FNRS at the IRIDIA laboratory in the Université Libre de Bruxelles (ULB), Brussels, Belgium. Dr López-Ibáñez has published more than 90 papers in international peer-reviewed conferences and journals. He is an elected member of the ACM SIGEVO Executive Board, Editor-in-Chief of *ACM Transactions on Evolutionary Learning and Optimization* and Associate Editor of the *Evolutionary Computation* journal. http://lopez-ibanez.eu



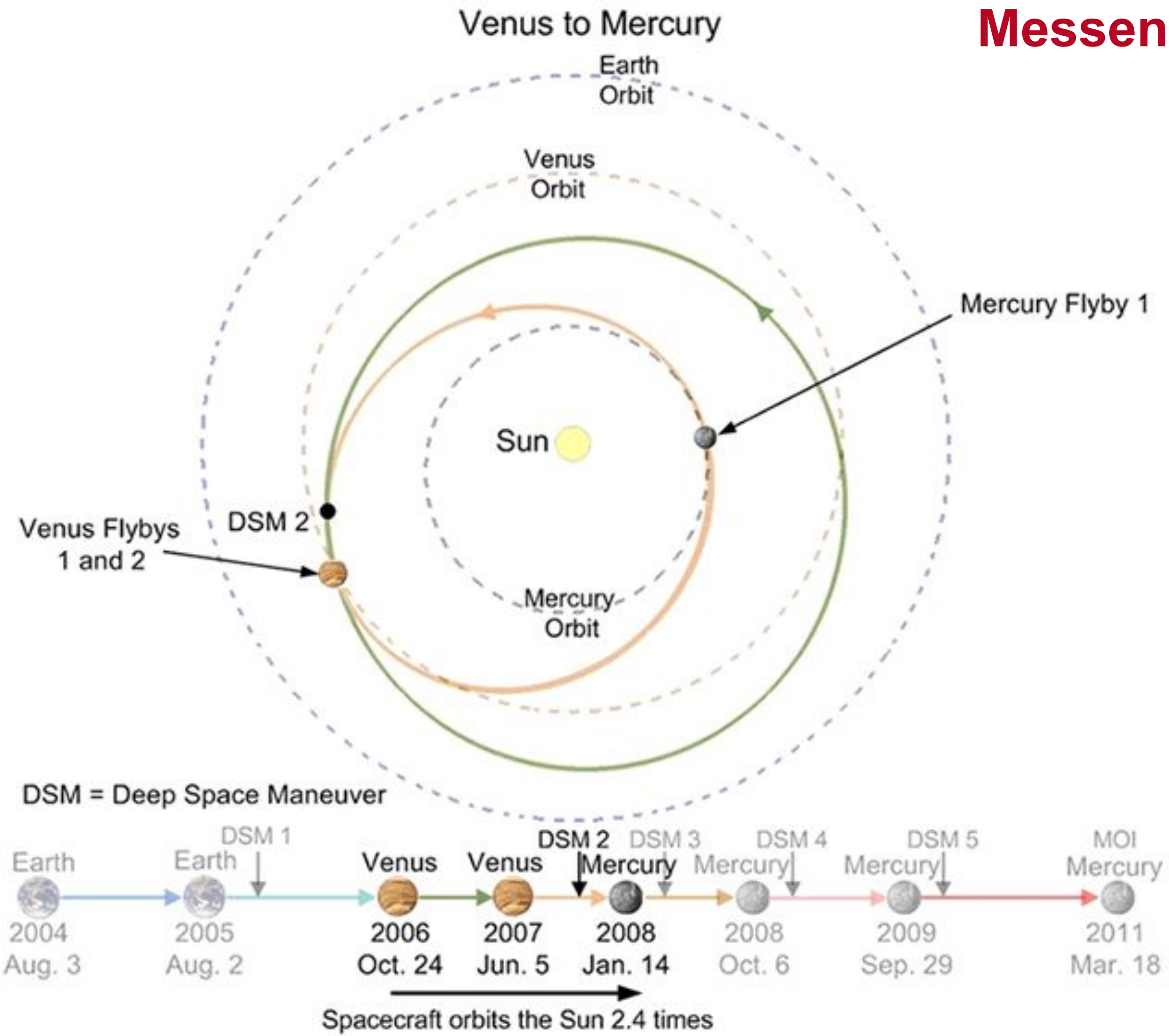

# Interplanetary Trajectories

## a. Real Missions

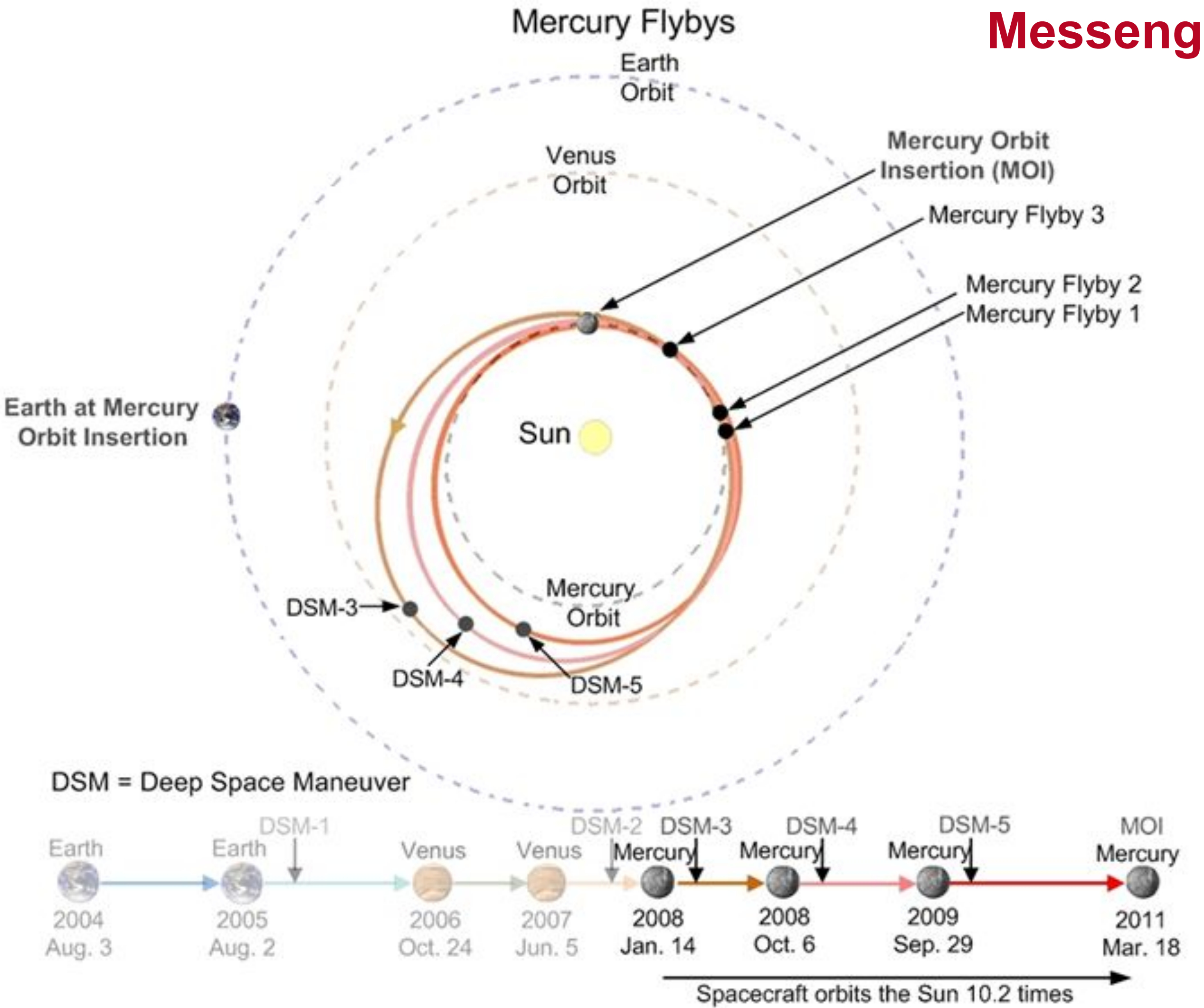
# Messenger (NASA)



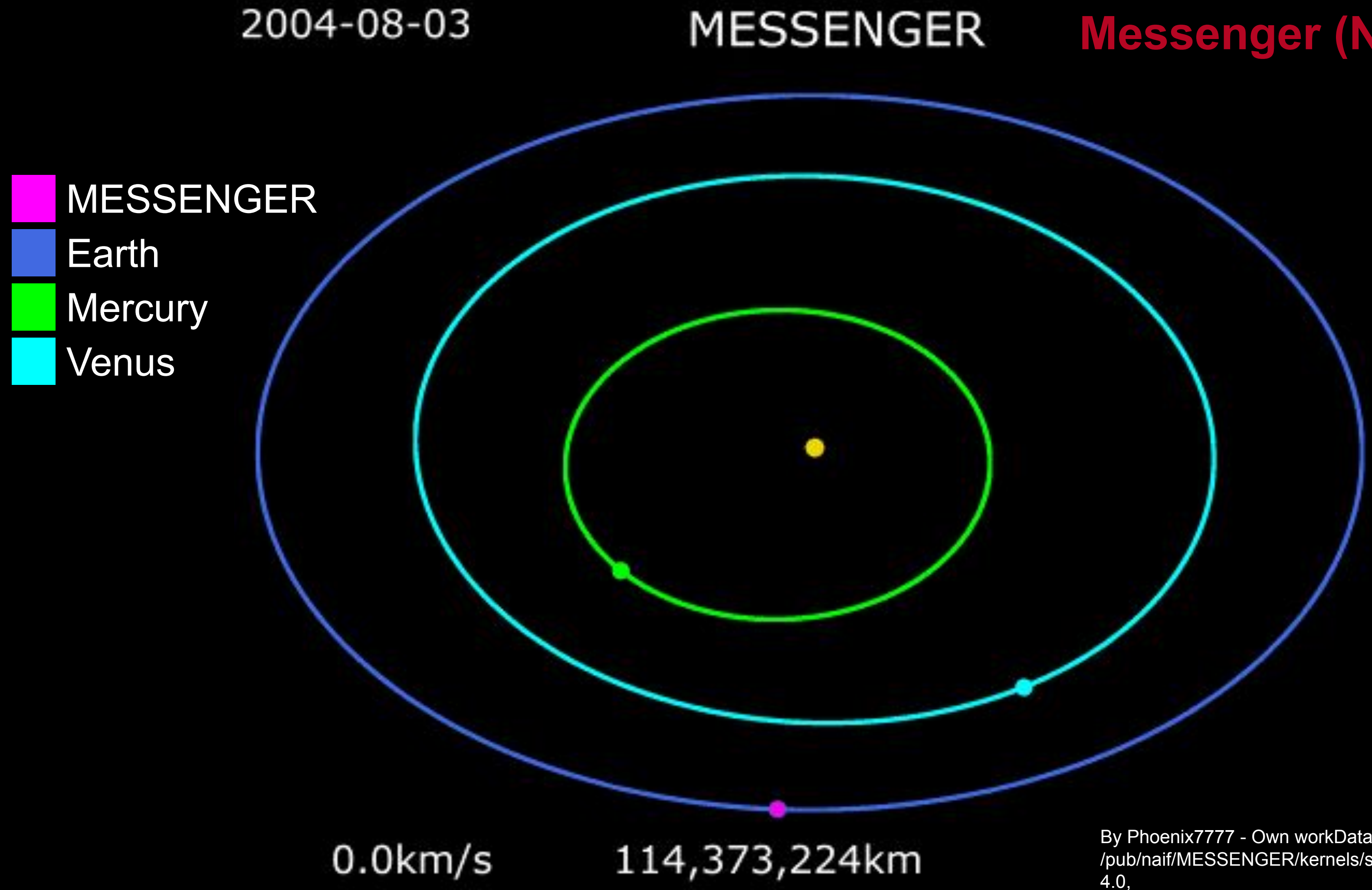
# Messenger (NASA)



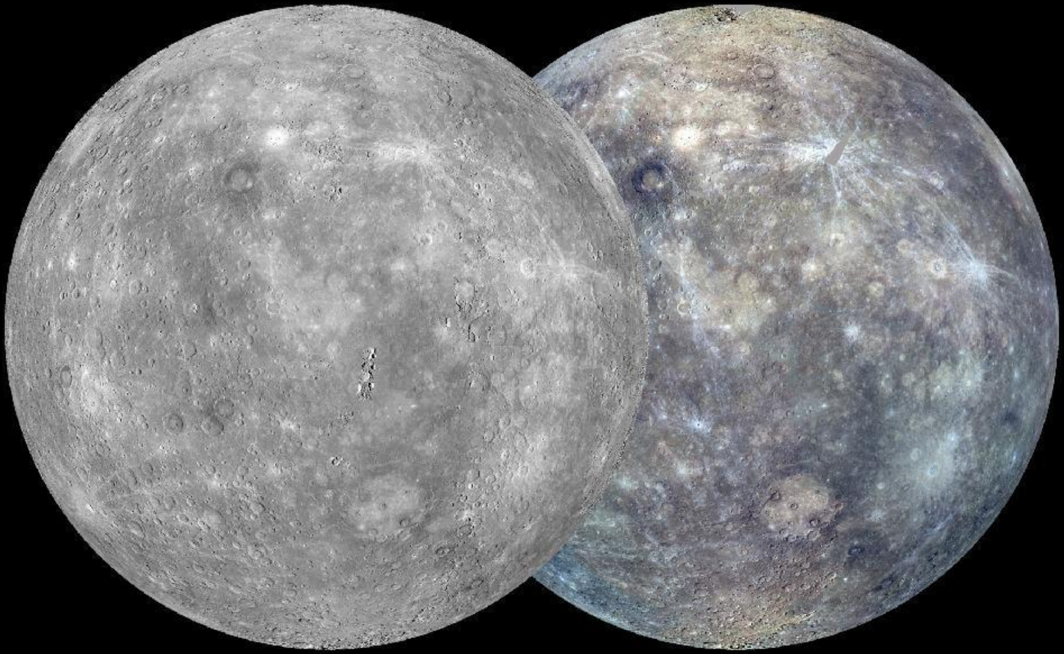
# Messenger (NASA)



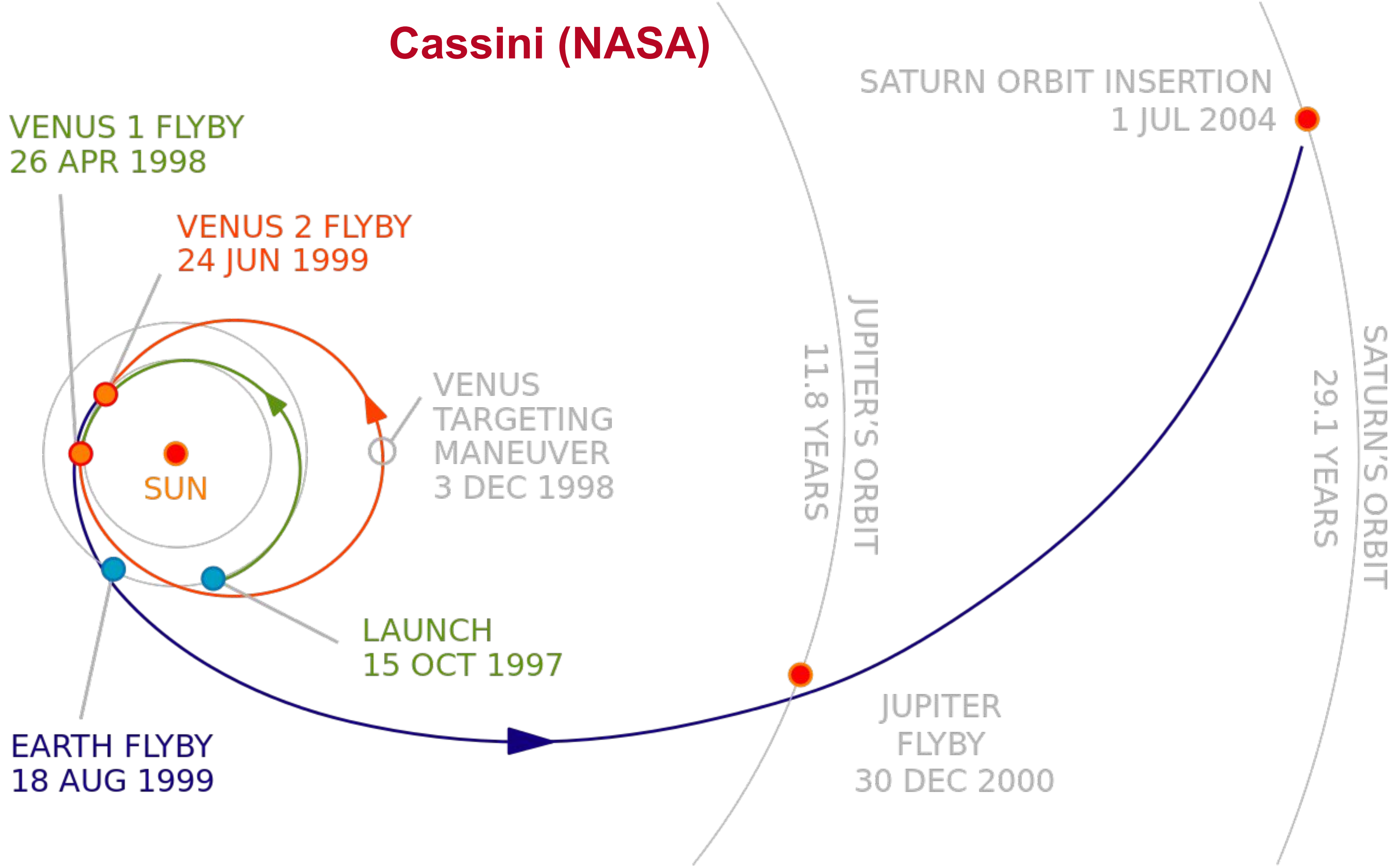
# Messenger (NASA)



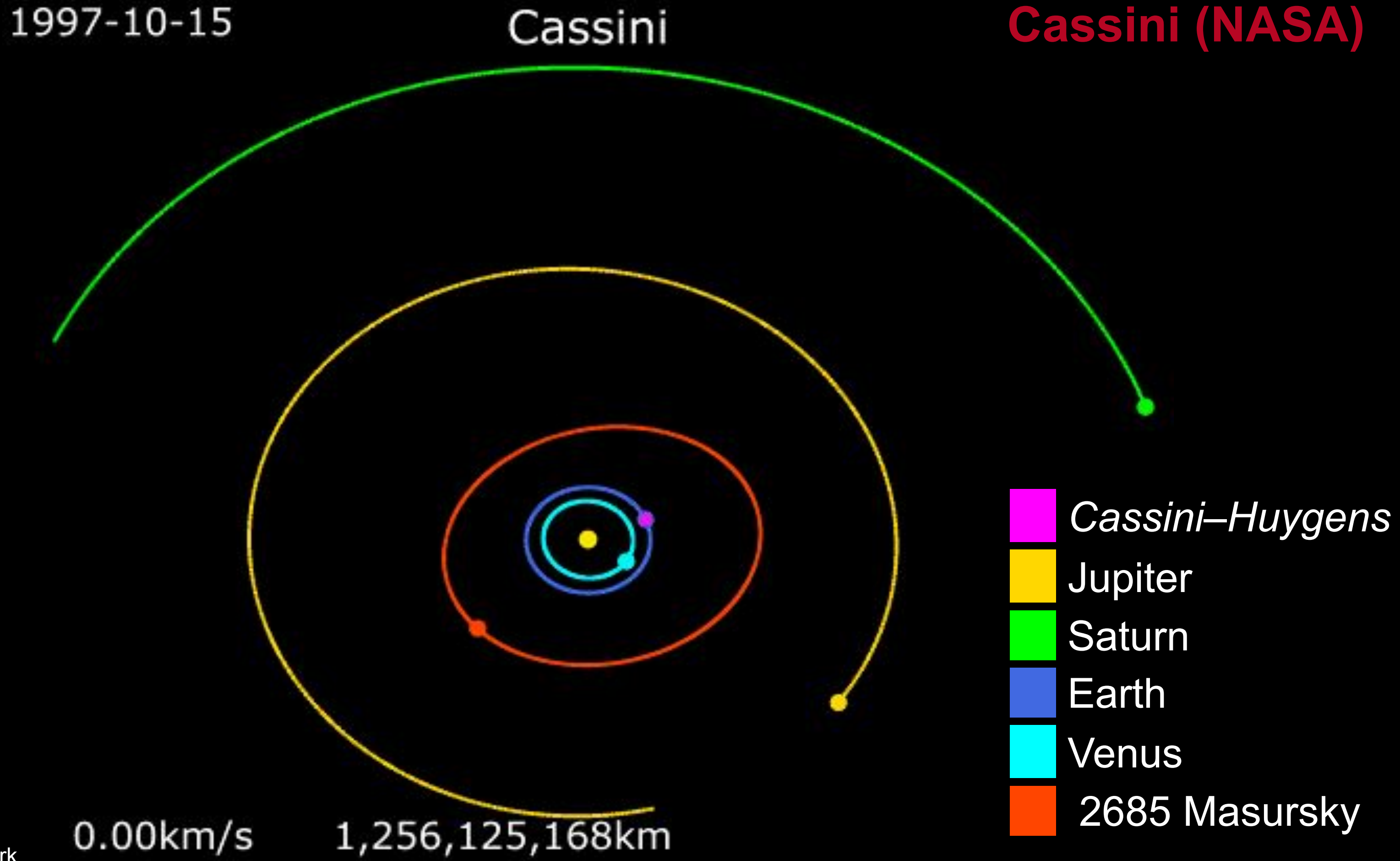
# Messenger (NASA)



# Cassini (NASA)

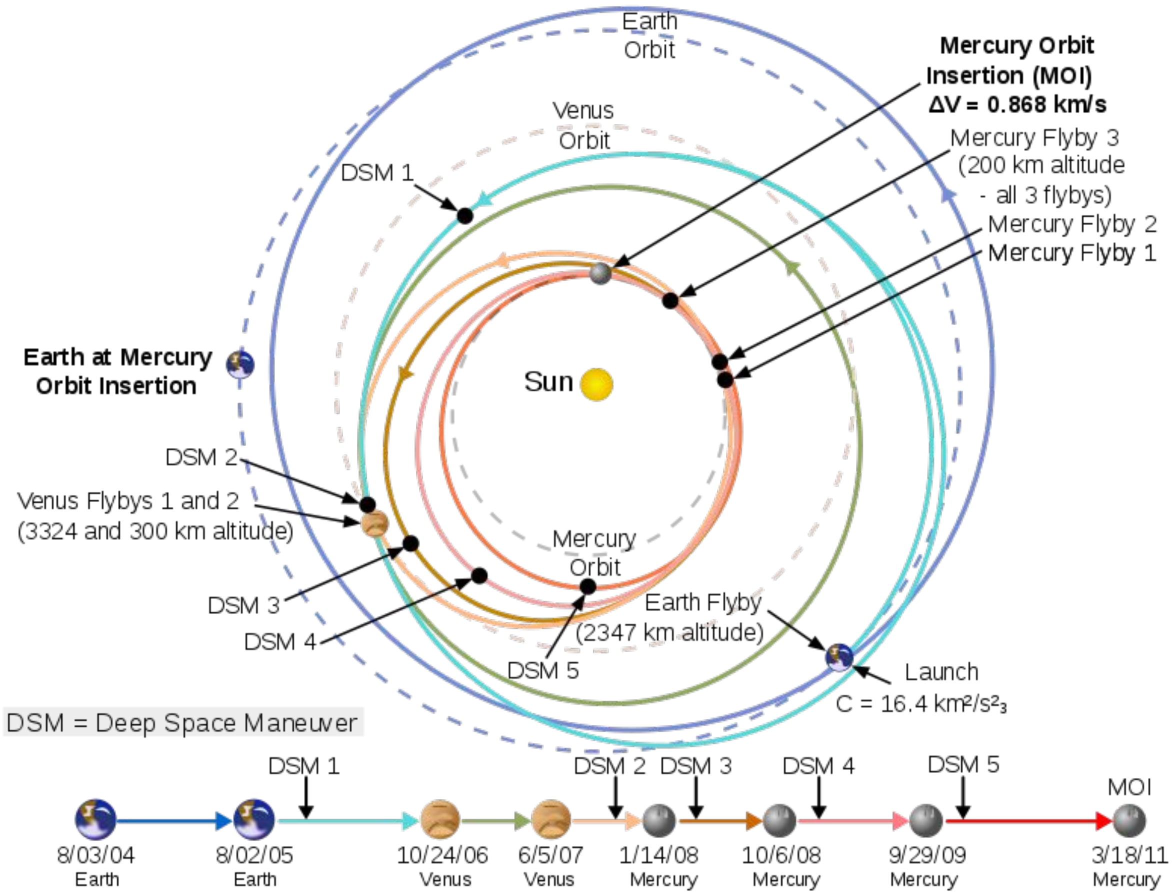


# Cassini (NASA)

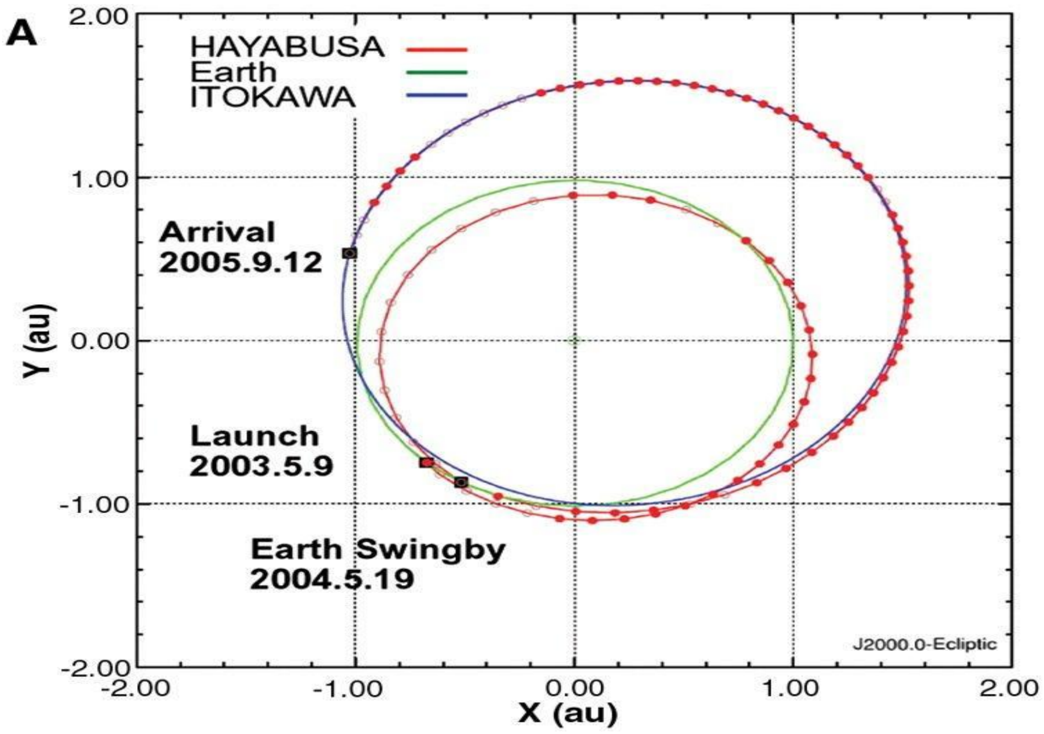



Cassini (NASA)
Water in Enceladus?

# Messenger (NASA)



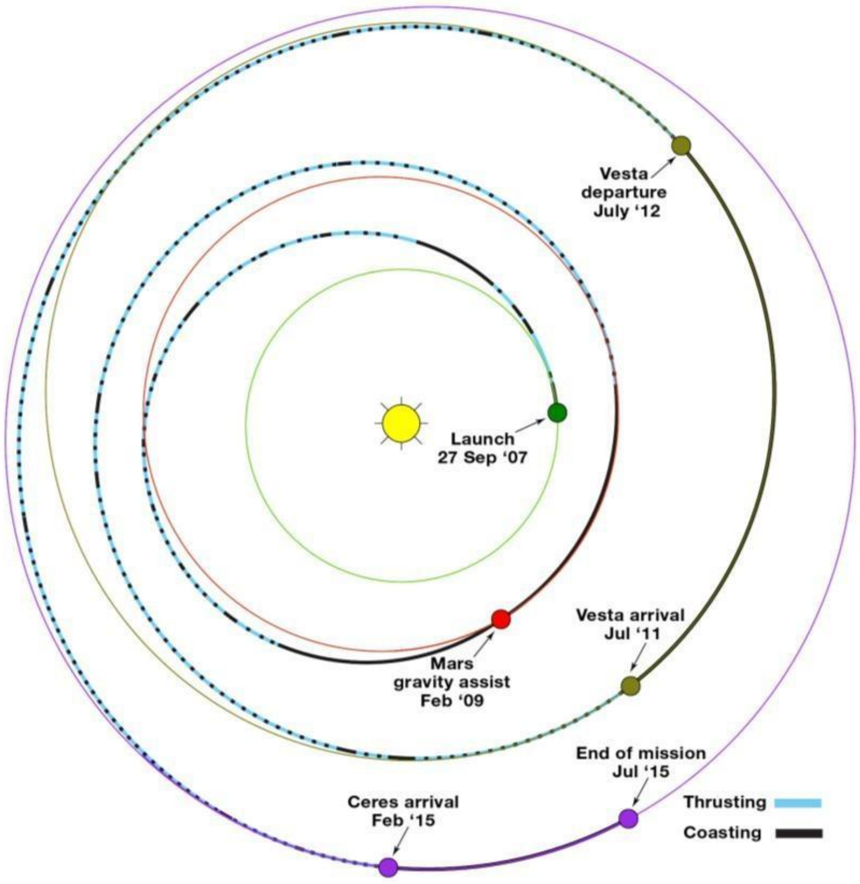
# Hyabusa (JAXA)



## Hyabusa (JAXA)

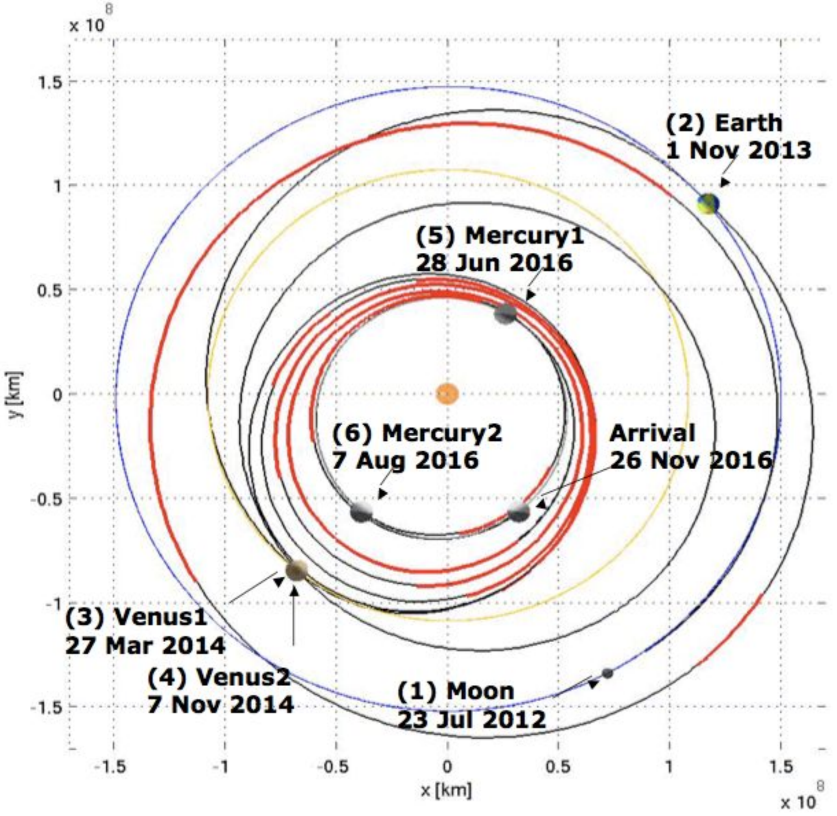
Rubble piles exist!

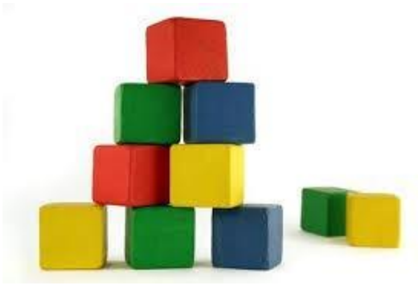
# Dawn (NASA)


Vesta departure July '12
Launch 27 Sep '07
Vesta arrival Jul '11
Mars gravity assist Feb '09
End of mission Jul '15
Ceres arrival Feb '15
Thrusting
Coasting

# Bepi Colombo (ESA)

Online Viewer

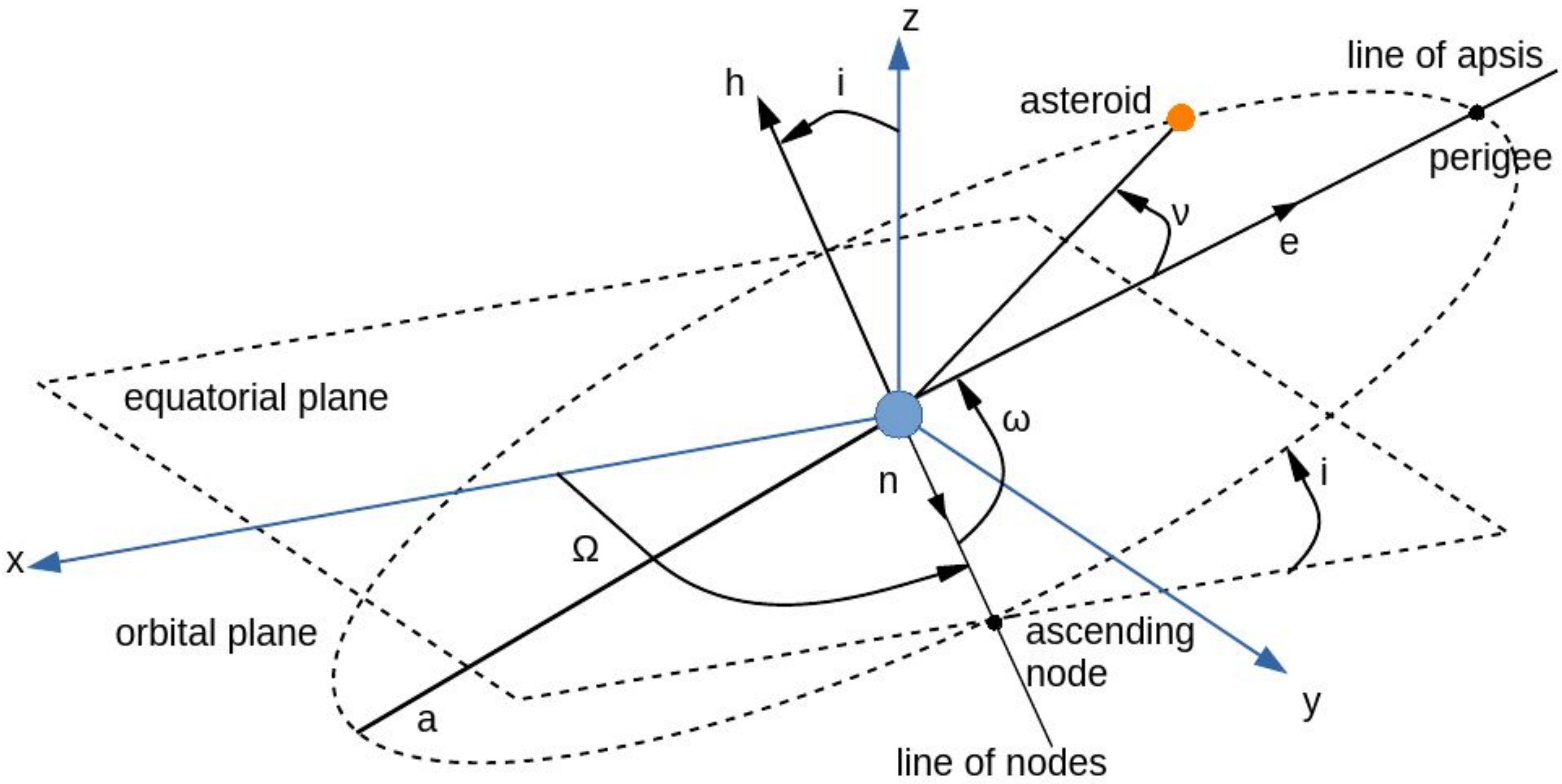




## Building blocks

1. Moving for free: Lagrange propagation
2. Changing orbits: Lambert's Problem
3. Change speed (almost) for free: Mivovitch Fly-bys (slingshot manoeuvre)

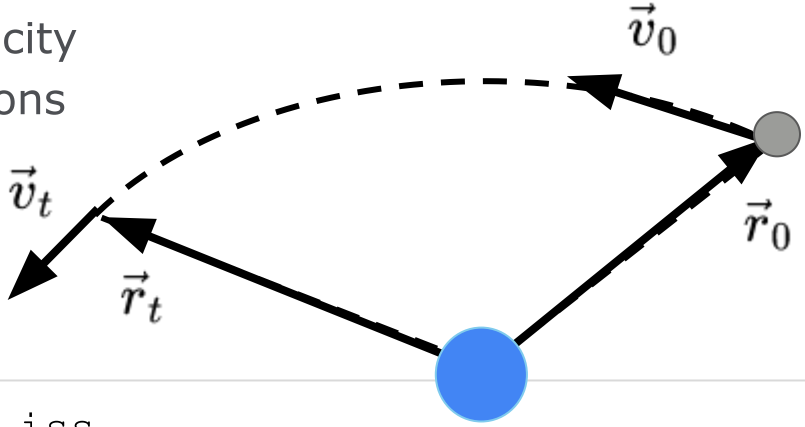
# Moving for free: Orbital (Lagrange) Propagation



# Orbital (Lagrange) Propagation
# (moving without consuming fuel)

- We can "propagate" a body to any time in the future i.e., predict the future position and velocity of a celestial body from starting conditions

- Basic operation in various software packages

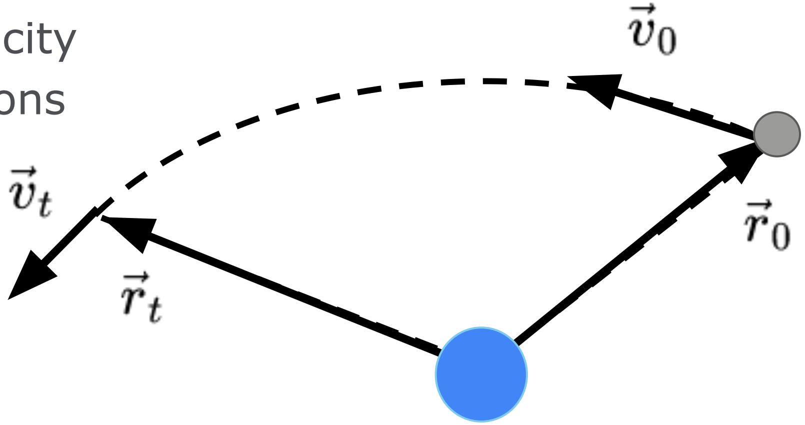



```
In [1]: from poliastro.examples import iss
In [2]: iss
Out[2]: … orbit around Earth (♁) at epoch 2013-03-18 12:00:00.000 (UTC)
In [4]: iss.rv()
Out[4]: ([859.07, -4137.20, 5295.56] km, [7.37, 2.08, 0.44] km/s)
In [5]: iss.propagate(30 << u.day).rv()
Out[5]: ([1568.72, 4533.24, -4803.90] km, [-7.20, -0.24, -2.57] km/s)
```

# Orbital (Lagrange) Propagation
## (moving without consuming fuel)

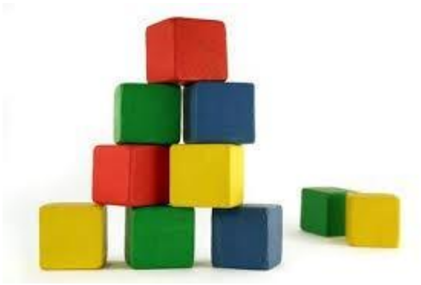
- We can "propagate" a body to any time in the future i.e., predict the future position and velocity of a celestial body from starting conditions

- Basic operation in various software packages




```python
import pykep as pk
import numpy as np
r,v = pk.propagate_lagrangian(r0 = [1,0,0], v0 = [0,1,0], tof = np.pi/2, mu = 1)
```

4.87 µs ± 59.4 ns per loop (mean ± std. dev. of 7 runs, 100000 loops each)



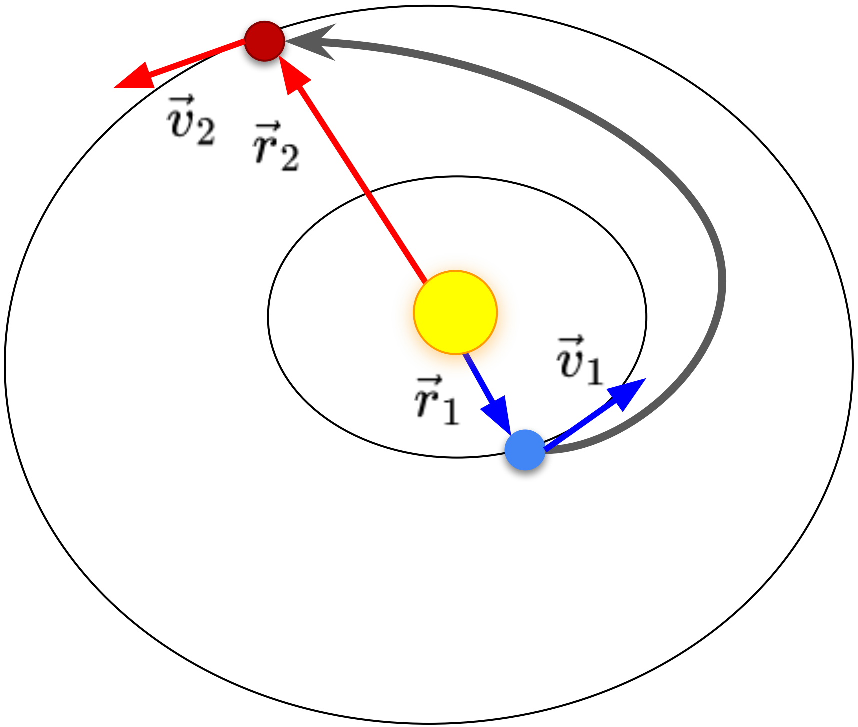
# Building blocks

2. Changing orbits: Lambert’s Problem

# How to visit a celestial body A with our spacecraft *S* ?

| | |
|---|---|
| $\tau$ | when we depart (epoch) |
| $t$ | how long we travel<br>($\tau + t$ = when we arrive) |
| $\mathbf{r}_1$ | point in orbit of *S* at epoch $\tau$ |
| $\mathbf{r}_2$ | point in orbit of *A* at epoch $\tau + t$ |
| $\Delta\mathbf{v}_1$ | impulse required at epoch $\tau$ to intercept *A* |
| $\Delta\mathbf{v}_2$ | impulse required at epoch $\tau + t$ to match orbit of A |

$$(\Delta\mathbf{v}_1, \Delta\mathbf{v}_2) = \text{Lambert}(S, A, \tau, t)$$



# How to visit a celestial body A with our spacecraft *S* ?

$\tau$ when we depart (epoch)

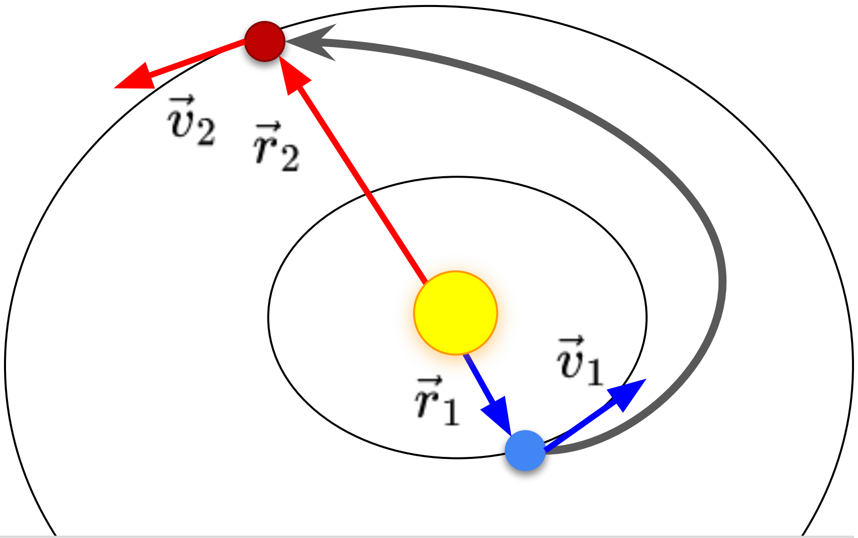
$t$ how long we travel
($\tau + t$ = when we arrive)

$\mathbf{r}_1$ point in orbit of *S* at epoch $\tau$

$\mathbf{r}_2$ point in orbit of *A* at epoch $\tau + t$

$\Delta\mathbf{v}_1$ impulse required at epoch $\tau$ to intercept *A*

$\Delta\mathbf{v}_2$ impulse required at epoch $\tau + t$ to match orbit of A



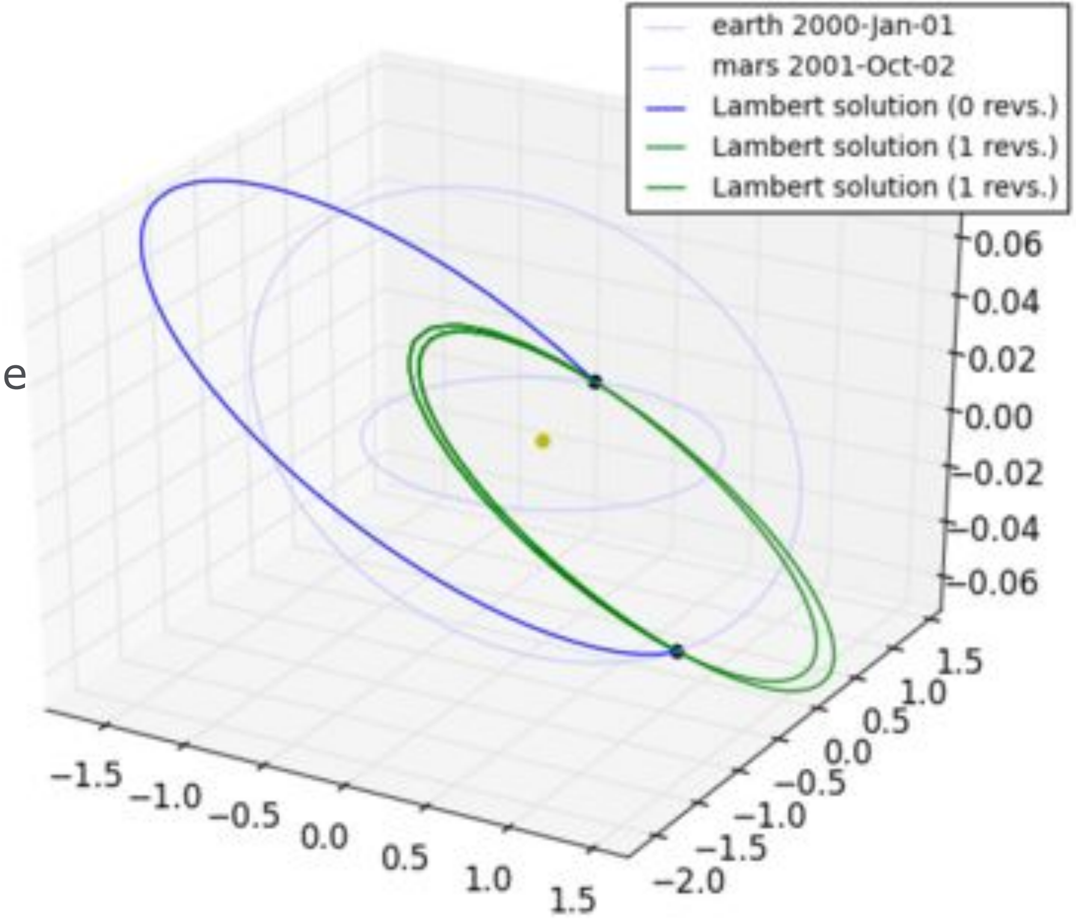

```
import pykep  as pk
import numpy as np
l = pk.lambert_problem(r1 = [1,0,0], r2 = [0,1,0], tof = np.pi/2, mu = 1., cw = False, max_revs = 0)
```

6.03 µs ± 169 ns per loop (mean ± std. dev. of 7 runs, 100000 loops each)

# Lambert's Problem

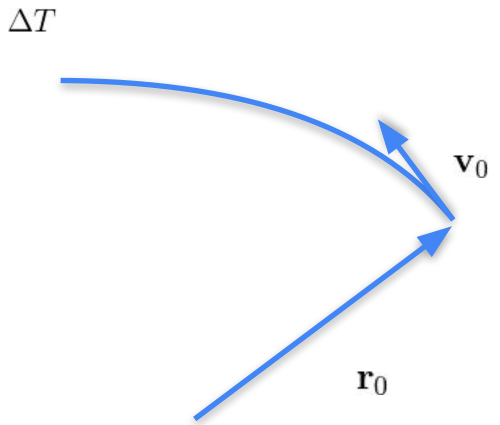
All Lambert problems have
1 single revolution solution

and may also have 2·N multiple revolution solutions
(if enough transfer time)

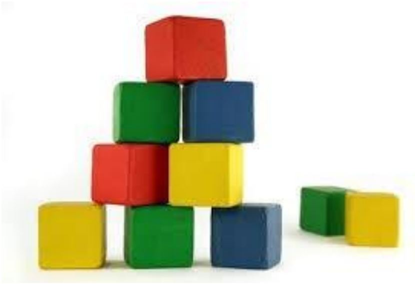


# Lagrange Propagation

1. Predicting the time evolution of an orbit from starting conditions
2. It is an initial value problem (Cauchy)
3. Its solution can be efficiently obtained in terms of the Lagrange coefficients F,G
4. Kepler's equation needs to be solved to invert the eccentric anomaly - time relation.



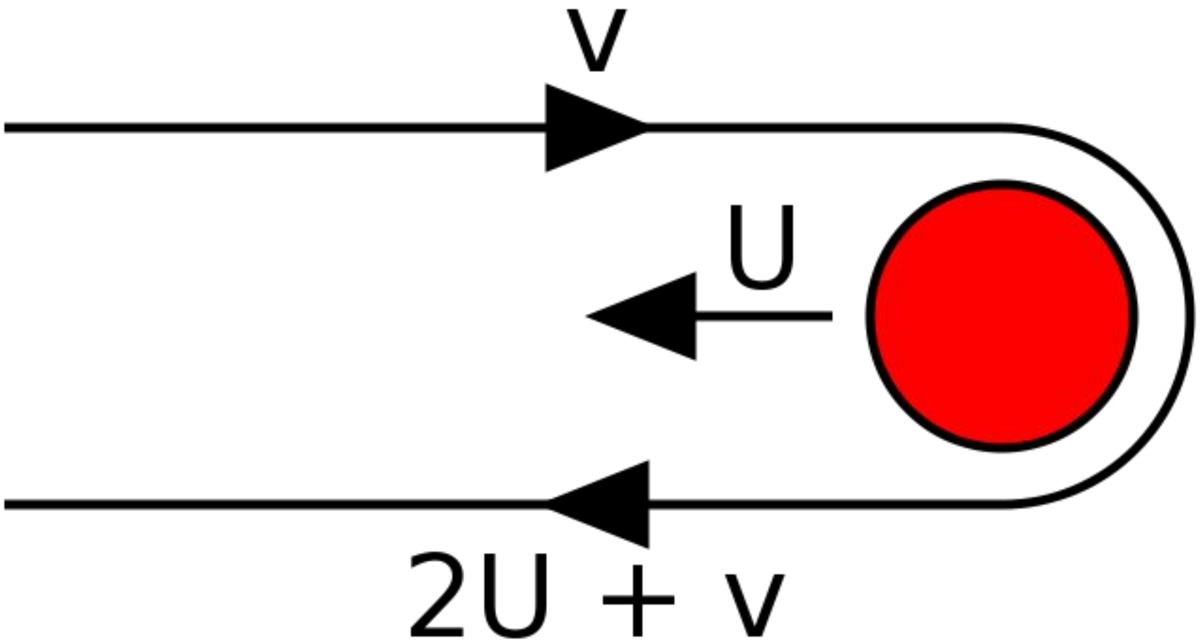

$$\begin{cases} \dot{\mathbf{r}} = \mathbf{v} \\ \dot{\mathbf{v}} = -\frac{\mu}{r^3}\mathbf{r} \\ \mathbf{r}(0) = \mathbf{r}_0 \\ \mathbf{v}(0) = \mathbf{v}_0 \end{cases}$$



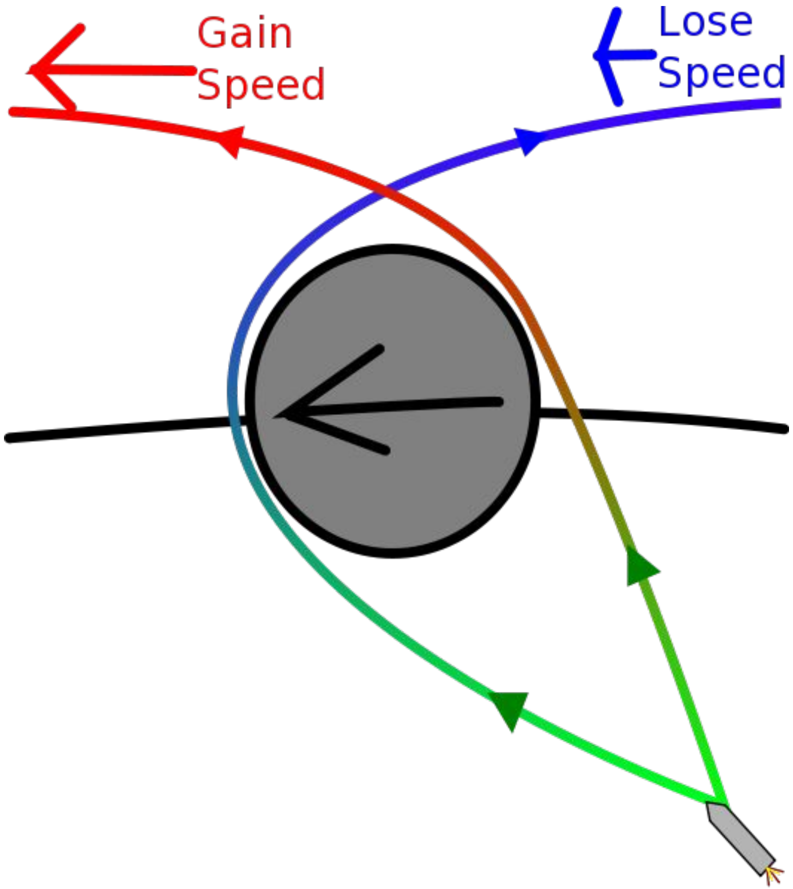
# Building blocks

3. Mivovitch Fly-bys (gravity assist, sling-shot maneuver)

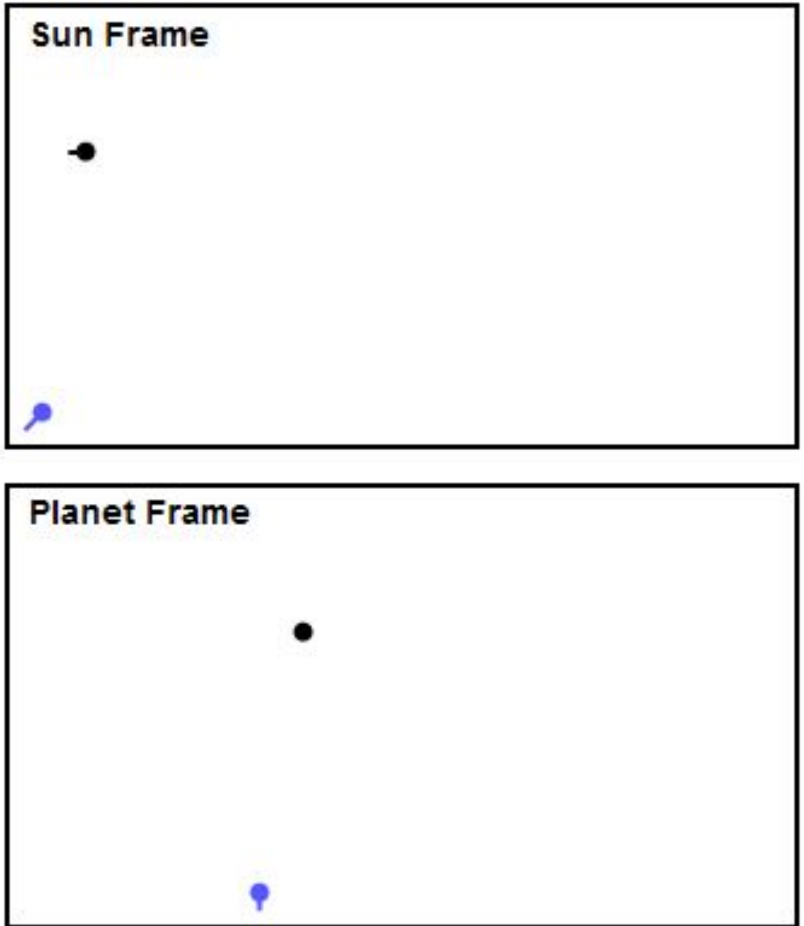
# Mivovitch sling-shot (gravity assist)



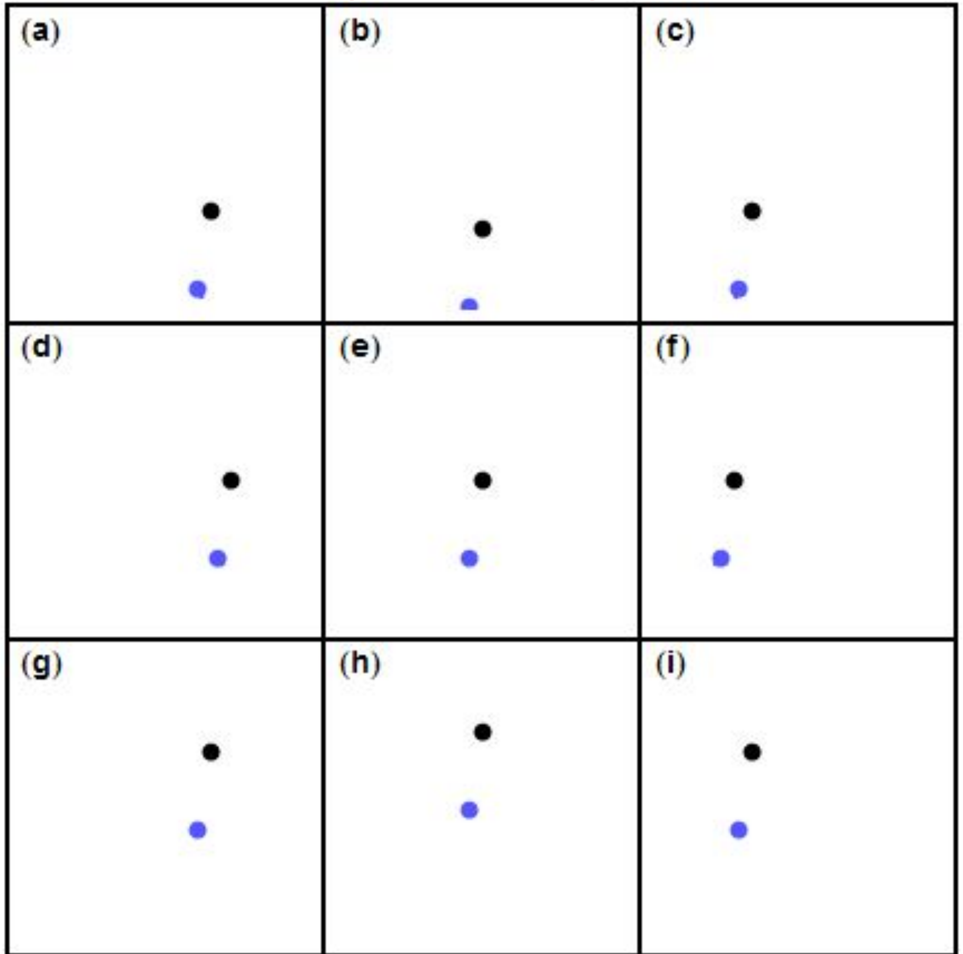
# Mivovitch sling-shot (gravity assist)



# Mivovitch sling-shot (gravity assist)



# Mivovitch sling-shot (gravity assist)

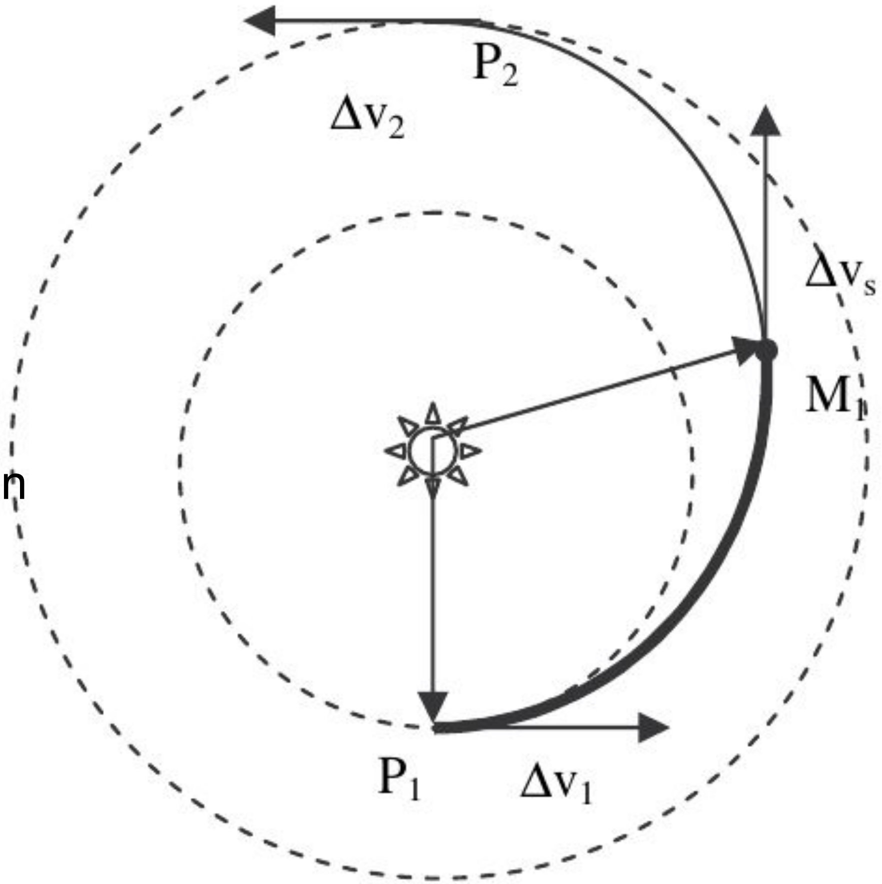


# Deep Space Maneuver (DSM)

- 3-impulse transfer

1. Lambert transfer between $P_1$ and $P_2$
2. At M1, do another Lambert transfer between $M_1$ and $P_2$

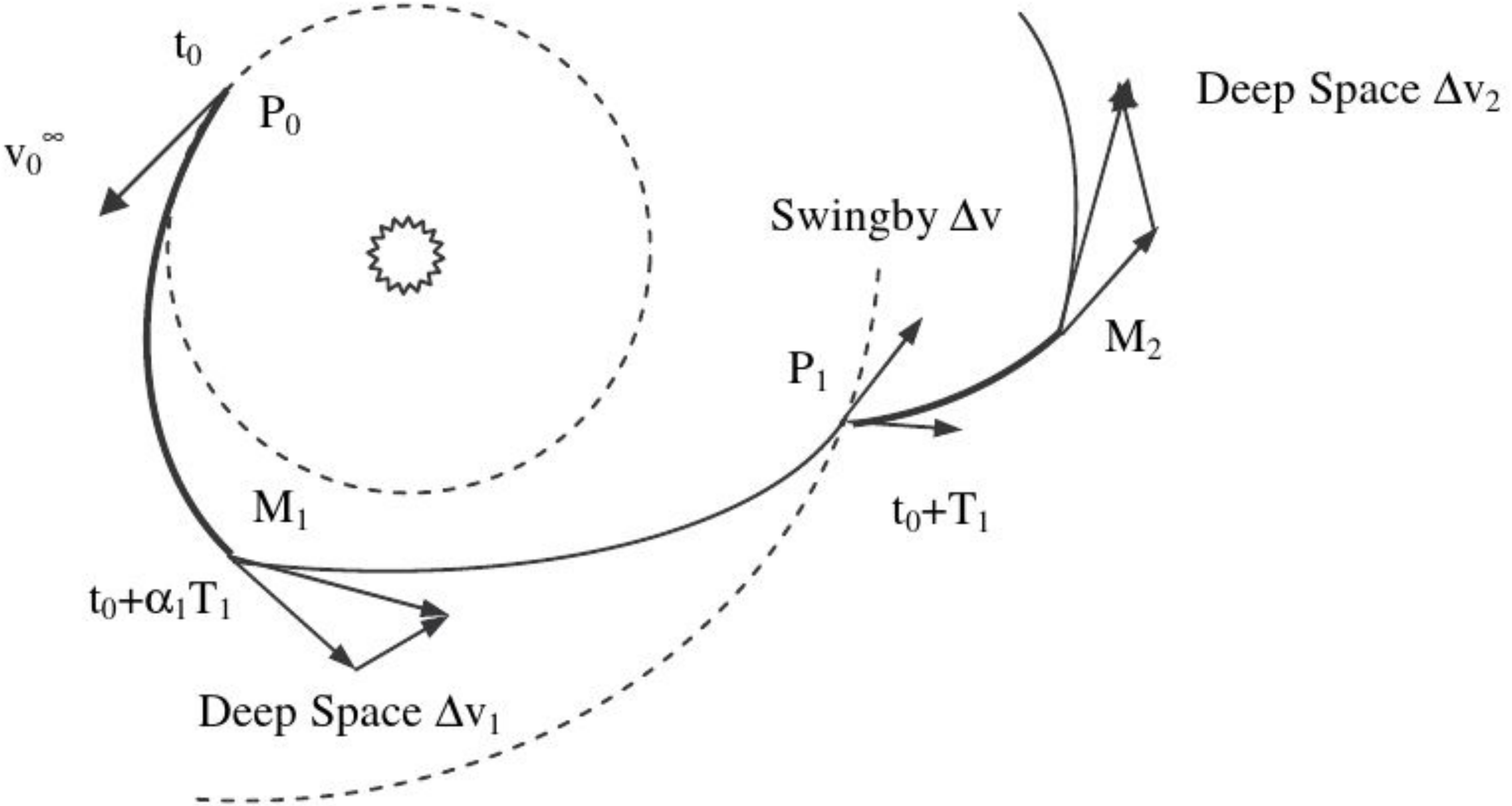
✅ More flexibility to reduce fuel consumption
❌ One more decision variable: time to $M_1$
❌ Twice the cost to evaluate





Vasile, Massimiliano, and Paolo De Pascale. "Preliminary design of multiple gravity-assist trajectories." *Journal of Spacecraft and Rockets* 43, no. 4 (2006): 794-805.

# Deep Space Maneuver (DSM)




Vasile, Massimiliano, and Paolo De Pascale. "Preliminary design of multiple gravity-assist trajectories." *Journal of Spacecraft and Rockets* 43, no. 4 (2006): 794-805.

# Asteroid Routing Problem


We are given:

- $n$ asteroids to visit $A = \{a_1, \ldots, a_n\}$
- Spacecraft departing from Earth and using only *impulsive* maneuvers

Minimize two objectives:

$$\Delta v = \sum_{i=1}^{2n} |\Delta \mathbf{v}_i| \qquad \text{(energy consumption, km/s)}$$

and

$$T = \sum_{i=1}^{2n} t_i \qquad \text{(elapsed time since } \tau_0\text{, days)}$$

# Asteroid Routing Problem

A solution $(\pi, \mathbf{t})$ where

- $\pi \in S_n$ permutation of the asteroids
- $\mathbf{t} = \{t_1, \ldots, t_{2n}\} \in \mathbb{R}^{2n}_{\geq 0}$ parking and transit times.

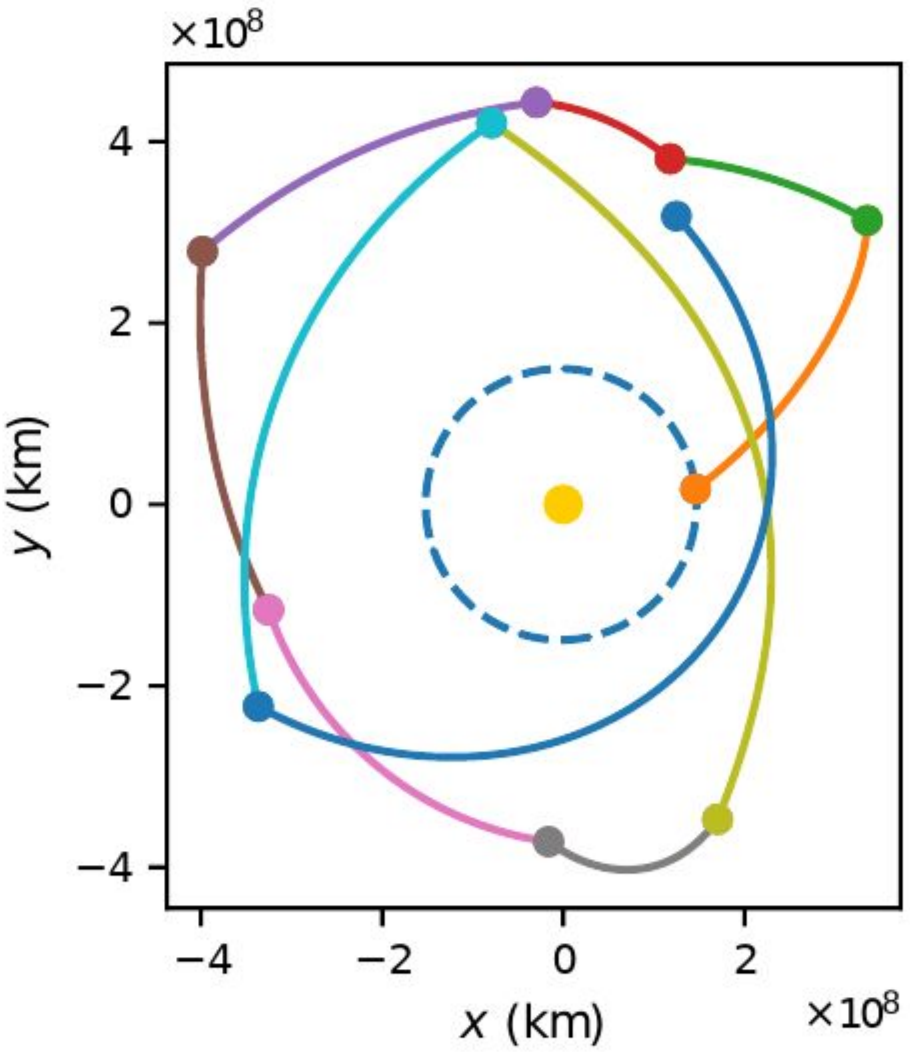
(1) $s$ launches from Earth at $\tau_0 + t_1$ to reach $a_{\pi(1)}$ in time $t_2$ (impulse $\Delta\mathbf{v}_1$)

(2) $s$ remains in $a_{\pi(1)}$ for time $t_3$ (impulse $\Delta\mathbf{v}_2$)

(3) $s$ launches to reach $a_{\pi(2)}$ in time $t_4$ (impulse $\Delta\mathbf{v}_3$)

(4) $s$ remains in $a_{\pi(2)}$ for time $t_5$ (impulse $\Delta\mathbf{v}_4$)

(5) ...



# Asteroid Routing Problem

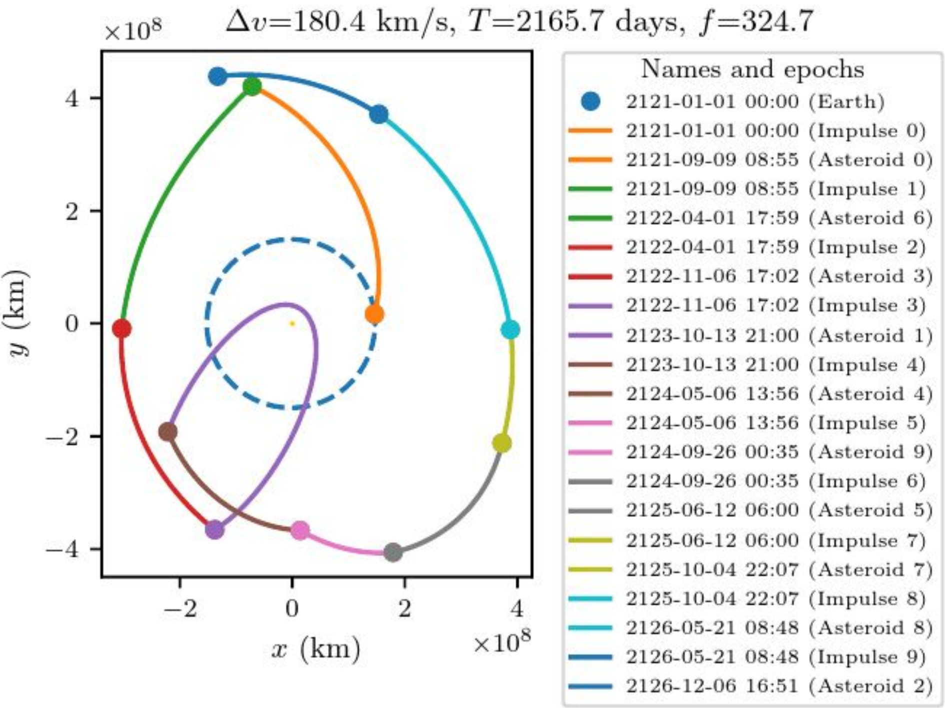
- Outer problem: Find the optimal permutation $\pi$ of the asteroids
- Inner problem: Find optimal vector **t** of times given $\pi$
  - Using 1000 iterations of deterministic SLSQP (Sequential Least Squares Programming) algorithm
  - Given the same $\pi$, SLSQP returns the same **t**

How to solve the outer problem?

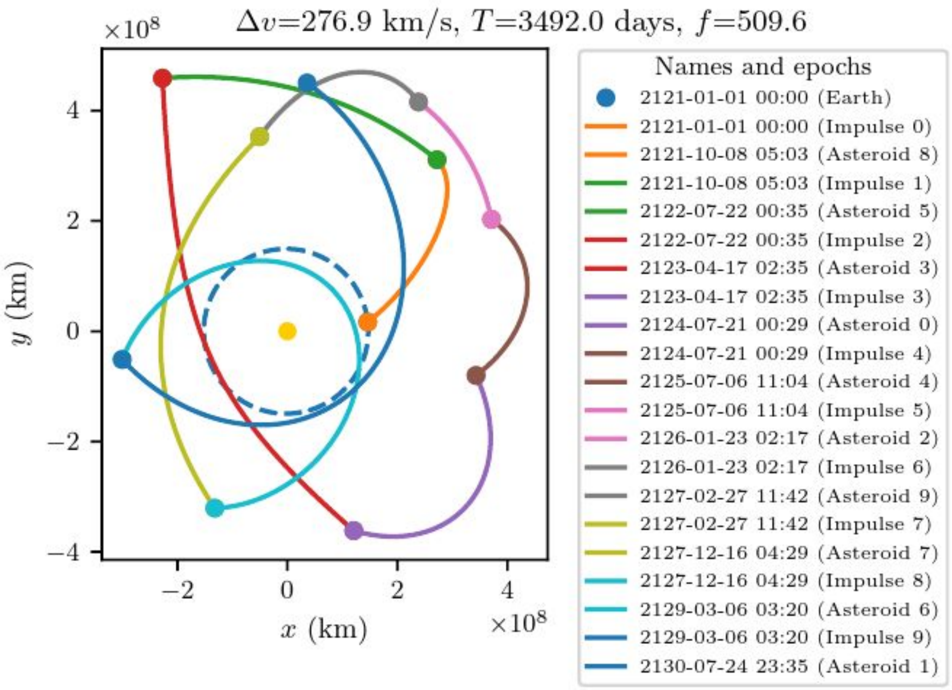
# Asteroid Routing Problem: Benchmark generator

- Generate random instances given *n* and *seed*
by sampling from 80 000 asteroids
- Data from the 11th Global Trajectory Optimisation Competition
https://gtoc11.nudt.edu.cn
- Astrophysics calculations (propagation, Lambert maneuver, etc.)
using poliastro (v0.16) (Cano Rodríguez et al., 2015)
- Solves the inner problem using Scipy's SLSQP
- Calculates objective functions for you

# Asteroid Routing Problem

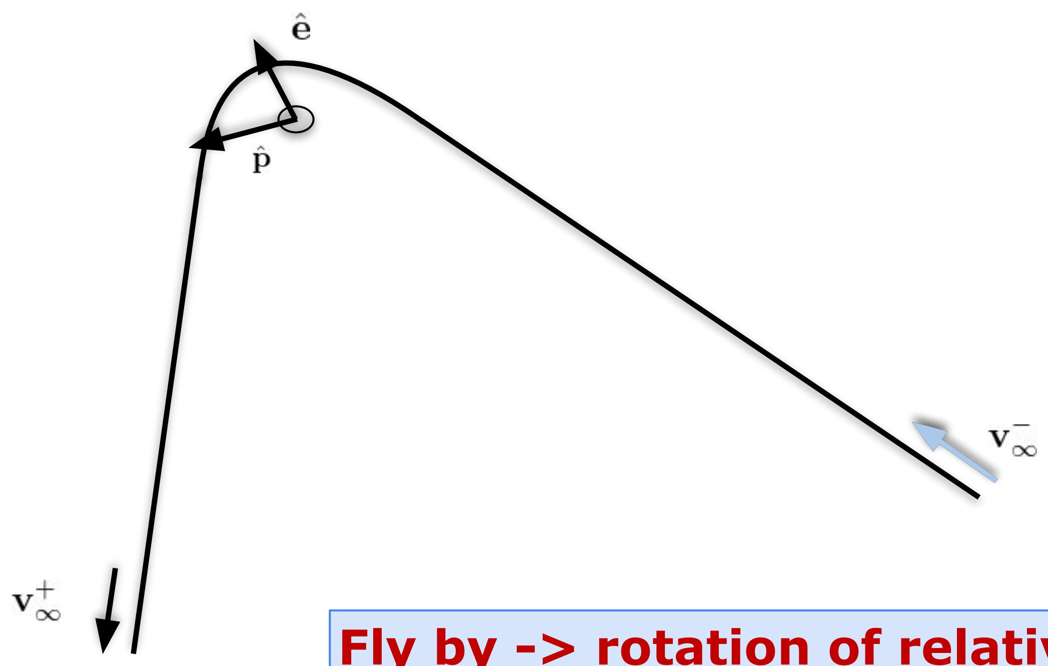



CEGO+Greedy
on ARP $n$ = 10 seed=73

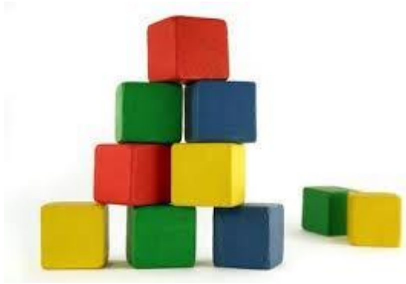



UMM+Greedy
on ARP $n$ = 10 seed=73

# Mivovitch sling-shot




**Fly by -> rotation of relative velocity**



# Building blocks

## iii. Mivovitch Fly-bys (planetary kick manoeuvre)

```
import pykep
import numpy as np
v2_eq, delta_ineq = pk.fb_con([1000,0,0], [900,440,0], pk.planet.jpl_lp('earth'))
```

3.17 µs ± 31.7 ns per loop (mean ± std. dev. of 7 runs, 100000 loops each)

```
import pykep
import numpy as np
vout = pk.fb_prop(v = [1,0,0],v_pla = [0,1,0], rp=2., beta=3.1415/2, mu_pla=1.)
```

3.47 µs ± 68.1 ns per loop (mean ± std. dev. of 7 runs, 100000 loops each)

# Optimization problems in Space

- MGA: Multiple Gravity Assist Interplanetary Trajectory
  - box-constrained, low dimension, possibly combinatorial, SO, MO
- MGA-1DSM: MGA where only one Deep Space Maneuver is allowed in each leg
  - box-constrained high dimension, possibly combinatorial, SO, MO
- MGA-LT: MGA Low-Thrust maneuvers
  - non linear constraint, high dimension, possibly combinatorial SO, MO
- Tours and multiple visits:
  - mainly combinatorial, similar to TSP variants, SO, MO

# MGA Model

Given a planetary sequence of N planets find:

$$x = [t_{dep}, T_1, .., T_N, T_{arr}]$$

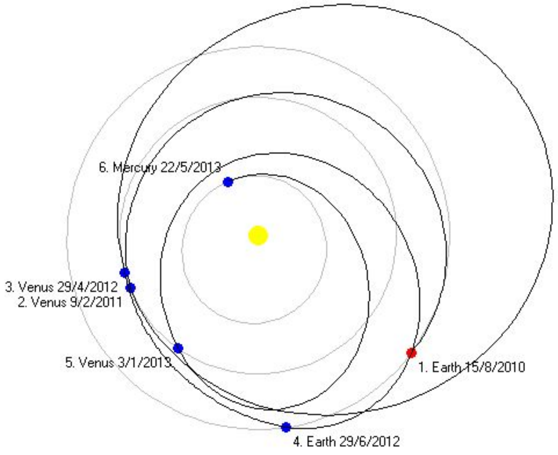
To minimise:

$$J = \Delta V_1 + \Delta V_2 + \ldots + \Delta V_{arr} \ (+\Delta V_{dep})$$

Subject to:

$$x \in [\underline{x}, \overline{x}]$$ Launch window constraint

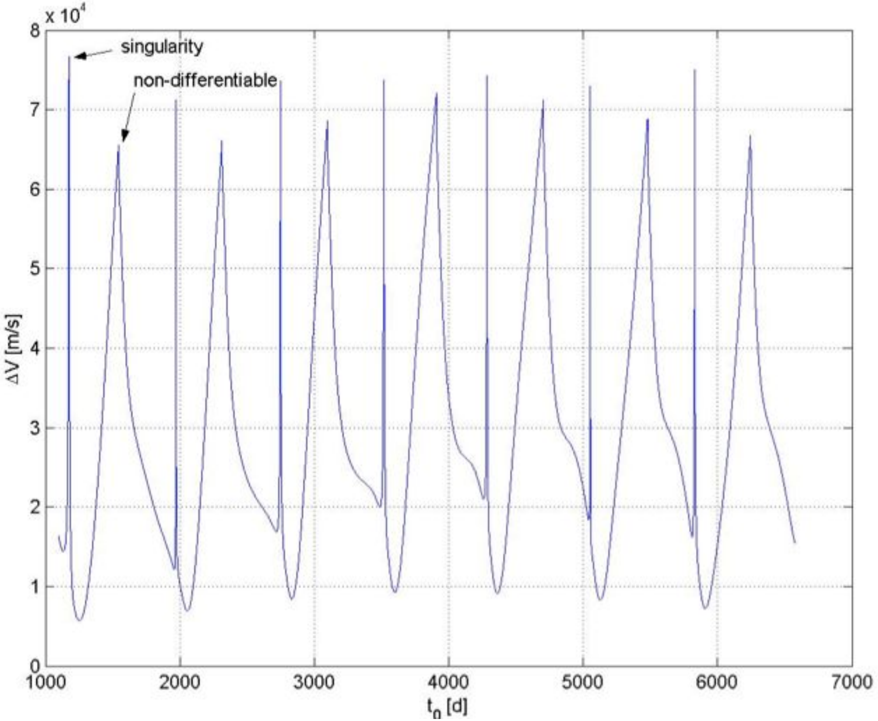
$$(\Delta V_{dep}^2 < C3_{launch})$$ Launcher constraint




Planetary sequence: EVVEVMe

# MGA: 1D case

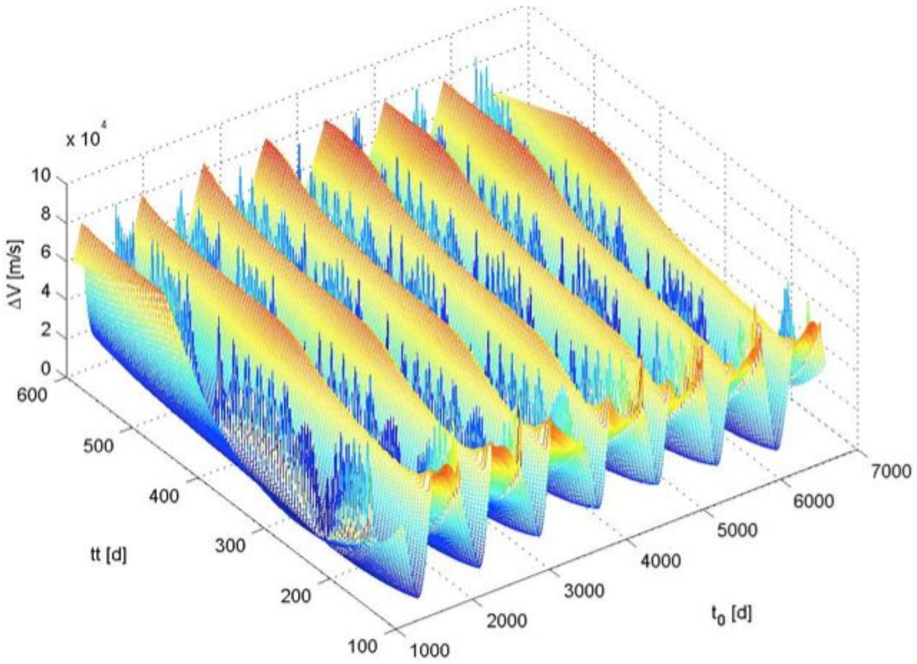
1. Earth-Mars transfer
2. Chemical propulsion
3. 200 days of transfer
4. MJD2000 used




Pierluigi Di Lizia and Gianmarco Radice. “Advanced Global Optimisation Tools for Mission Analysis and Design”, ACT Technical Report, 2004.

# MGA: 2D case

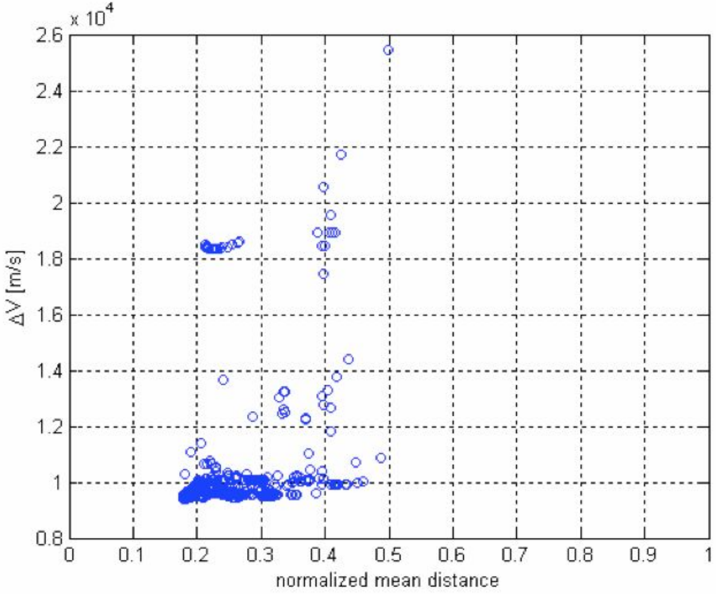
1. Earth-Mars transfer
2. Chemical propulsion
3. Days and MJD2000 used




Pierluigi Di Lizia and Gianmarco Radice. “Advanced Global Optimisation Tools for Mission Analysis and Design”, ACT Technical Report, 2004.

# MGA: 3D case

1. Earth-Jupiter-Saturn transfer
2. Local optima cluster together
3. Better local optima are close to the global one
4. Clustered local optima have similar objective values

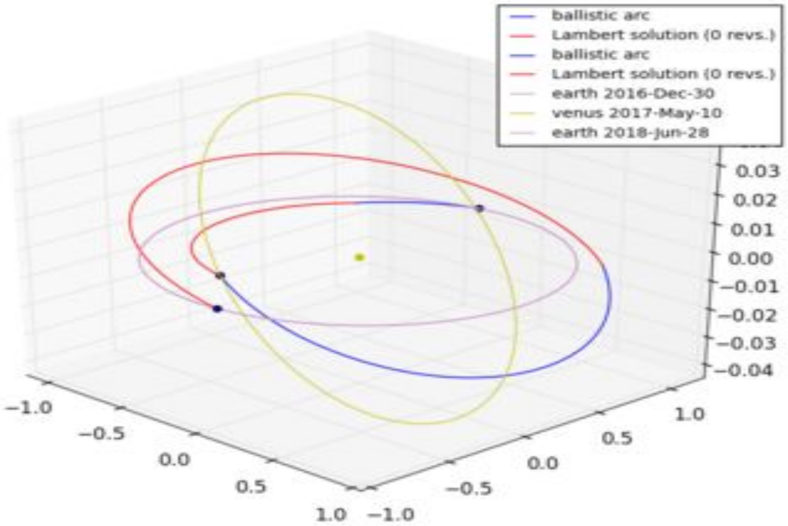


# MGA-1DSM: model

$$x = [t_0, V_\infty, u, v, \eta, T_0] + \ldots + [r_p, \beta, \eta_i, T_i]$$

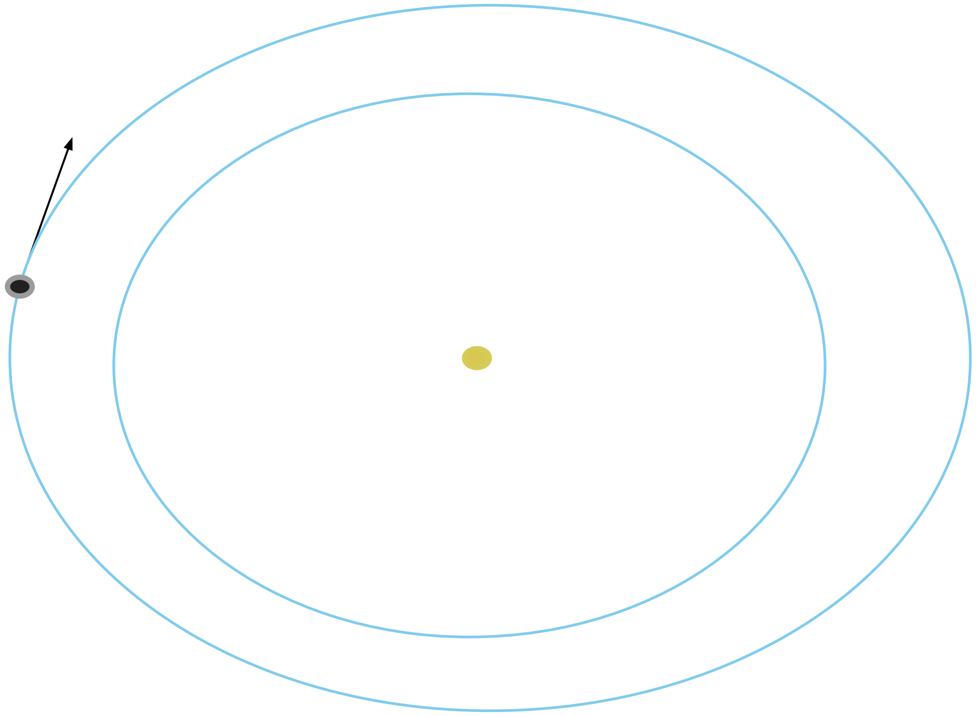
- Features of the MGA-1DSM model:
  - DSM value can be zero
  - Multi-revs are included
  - Resonant returns and backflips included

- Multiple objectives and combinatorial part possible: see SpOC: Trappist Tour



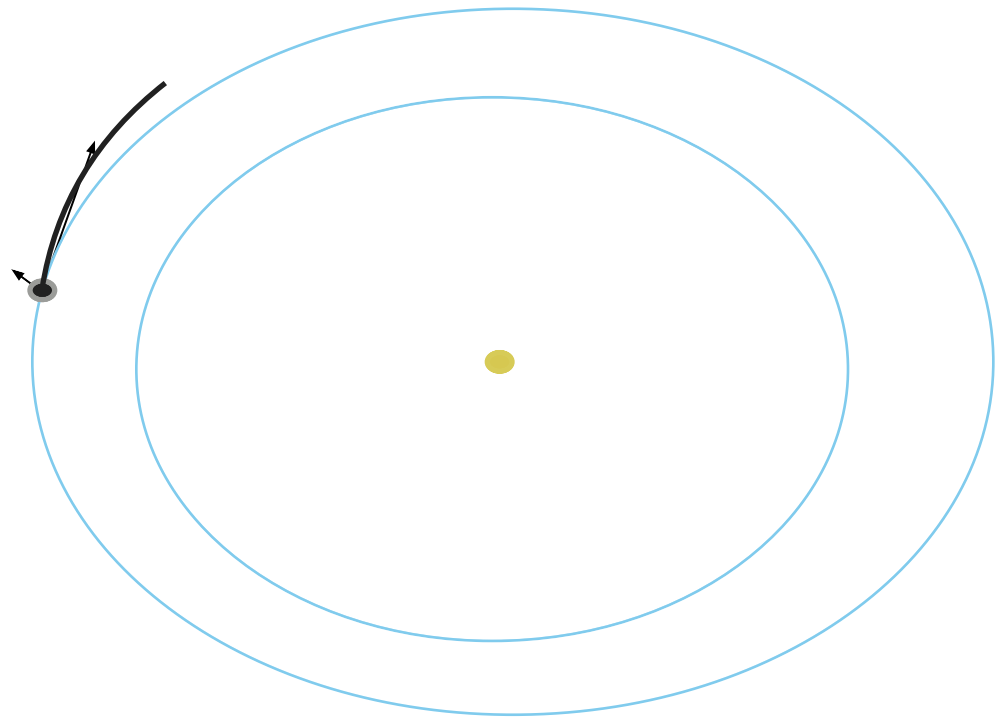
# MGA-LT: model

$t_0$



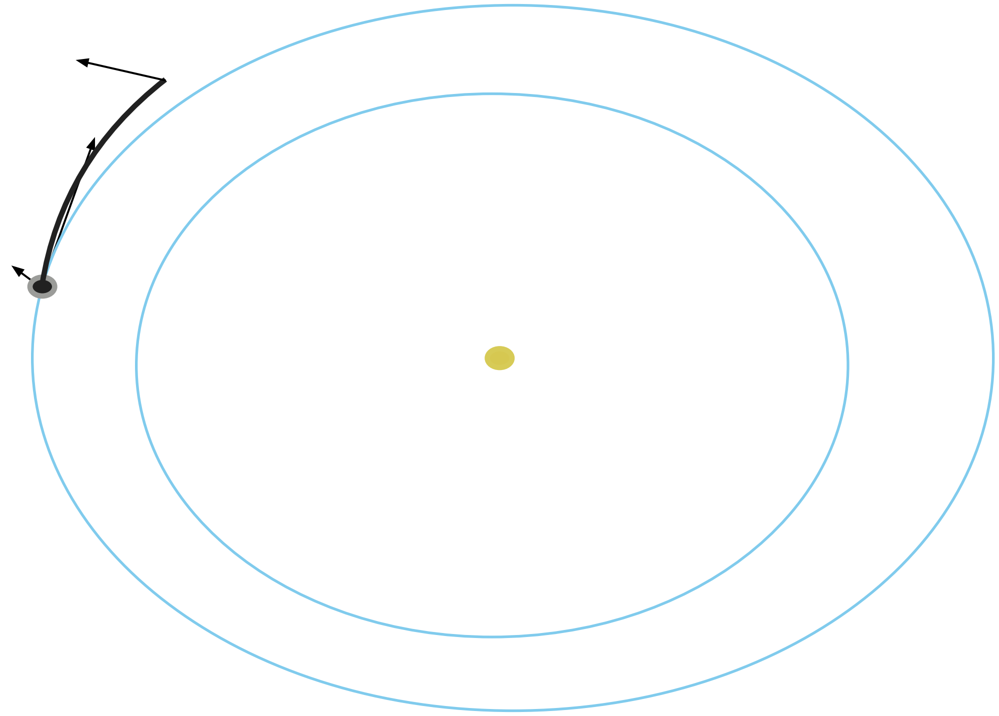
# MGA-LT: model

$t_0, \mathbf{v}_\infty$



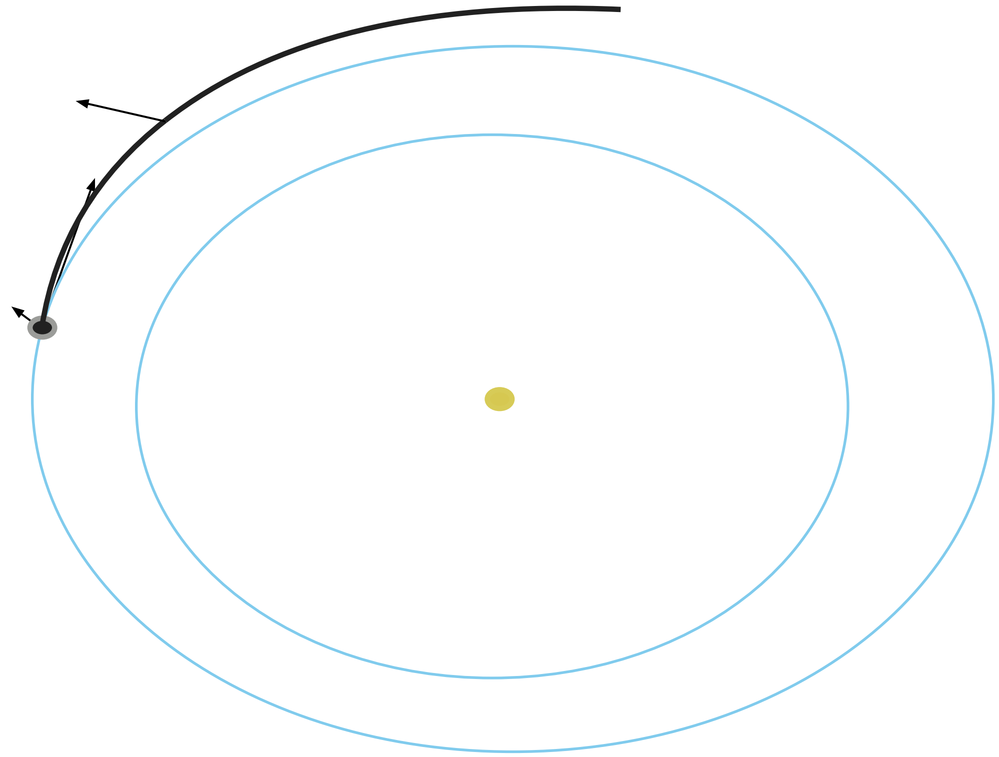
# MGA-LT: model

$t_0, \mathbf{v}_\infty, \Delta V_1$



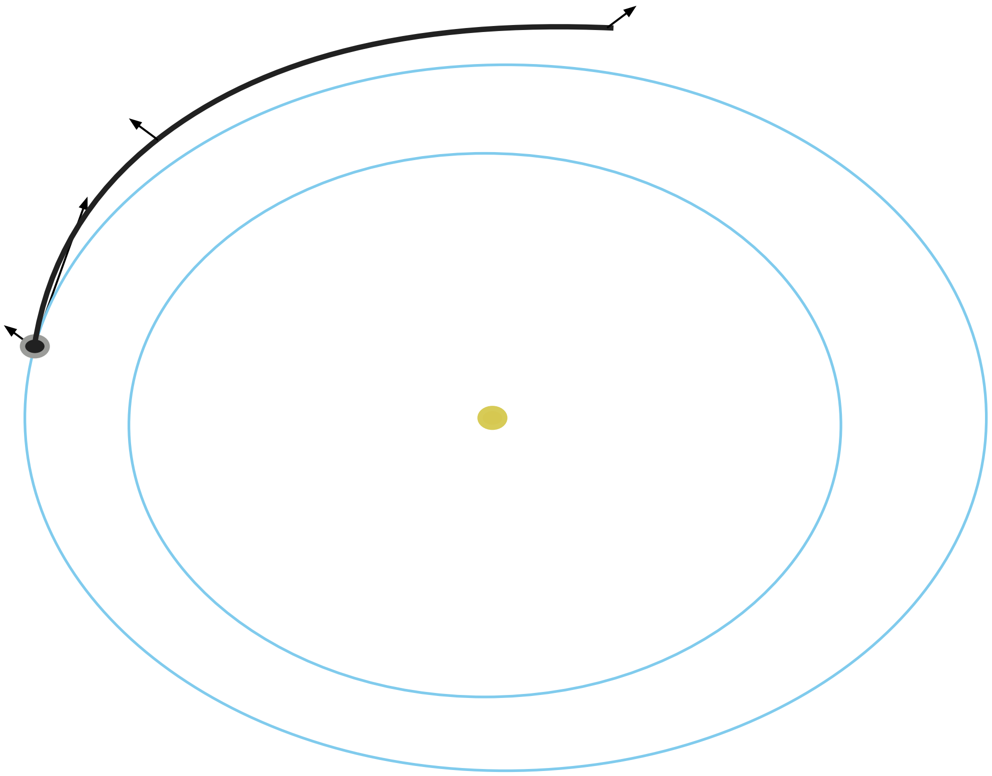
# MGA-LT: model

$t_0, \mathbf{v}_\infty, \Delta V_1$



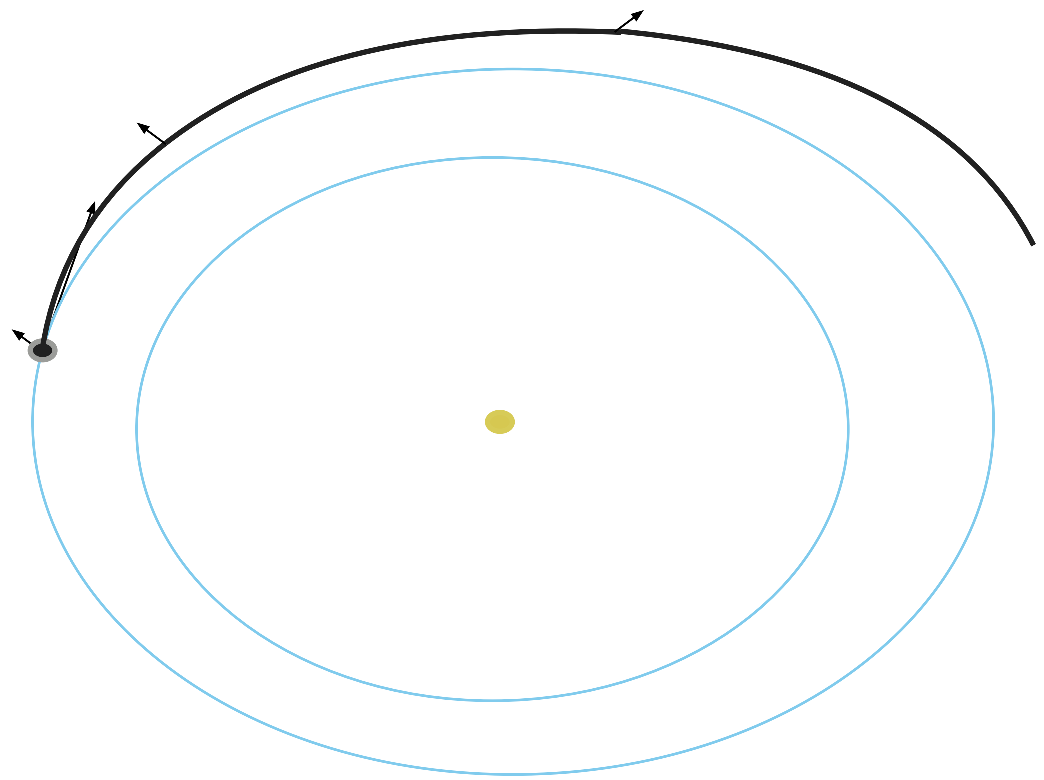
# MGA-LT: model

$t_0, \mathbf{v}_\infty, \Delta V_1, \Delta V_2$



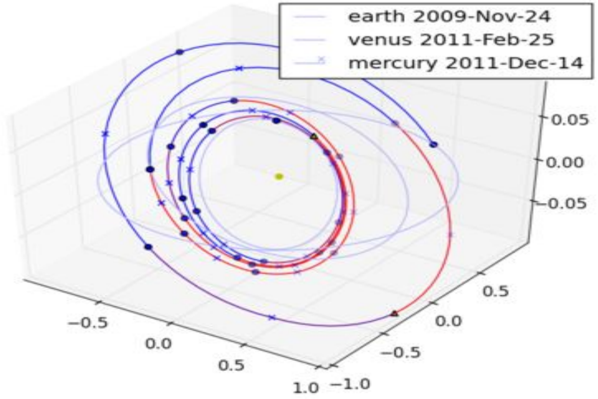
# MGA-LT: model

$t_0, \mathbf{v}_\infty, \Delta V_1, \Delta V_2$



# MGA-LT: model

$$x = [t0]$$
$$+[T_1, m_{f1}, V_{xi1}, V_{yi1}, V_{zi1}, V_{xf1}, V_{yf1}, V_{zf1}]$$
$$+[T_2, m_{f2}, V_{xi2}, V_{yi2}, V_{zi2}, V_{xf2}, V_{yf2}, V_{zf2}] + \ldots$$
$$+[u_x^1, u_y^1, u_z^1] + [u_x^2, u_y^2, u_z^2] + \ldots$$



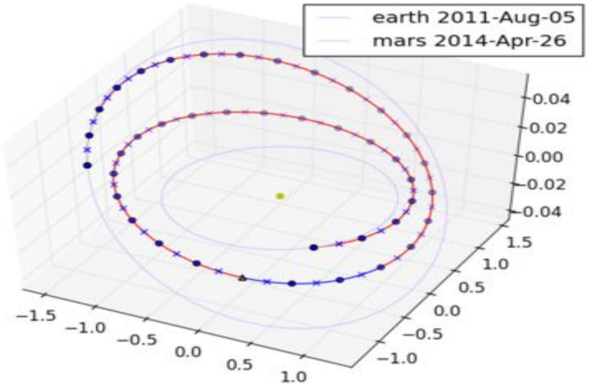

constraints:

mismatch

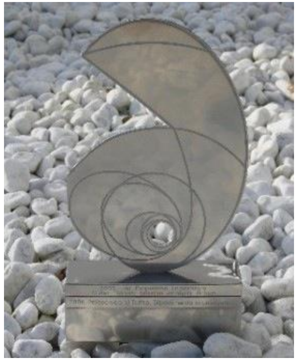
$|u^i|$

$|V_{f_i}| = |V_{f_{i+1}}|$

$V_{f_i} \cdot V_{f_{i+1}}| \geq \alpha$




- Features of the MGA-LT model:
    - Easy switch between low and high fidelity
    - Large convergence radius



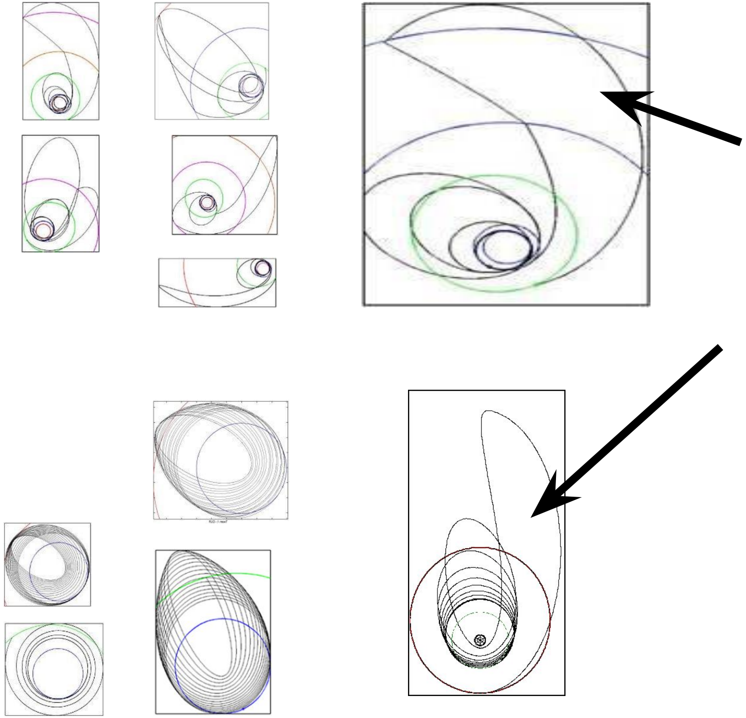
# Interplanetary Trajectories

## GTOC problems

# The America's cup of rocket science

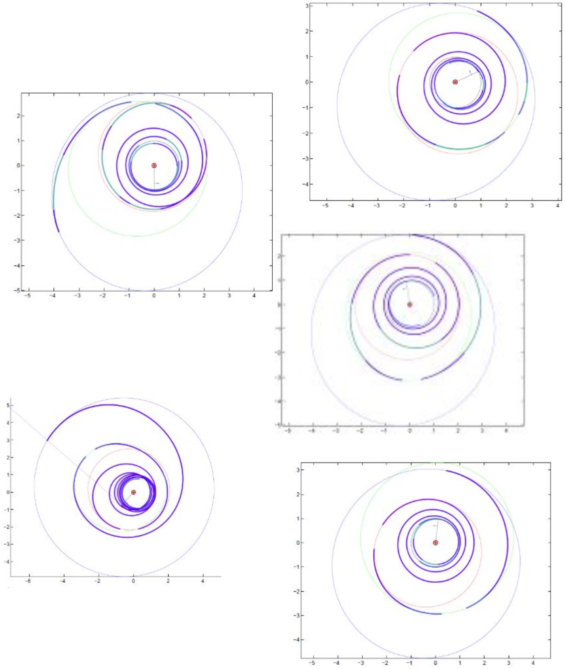
1. GTOC: Global Trajectory Optimization Competition
2. Taking place every year (roughly)
3. Near-to-impossible interplanetary trajectory problem: complexity ensures a clear competition winner
4. Open to academia, industry and space agencies
5. Winners organize and define the following edition
6. Creating a formidable database of challenging problems and solution methods
7. Competition duration is, usually, one month
8. The problem is rigorously defined so that solutions can be ranked with respect to a quantitative objective value

# GTOC 1: Save the Earth



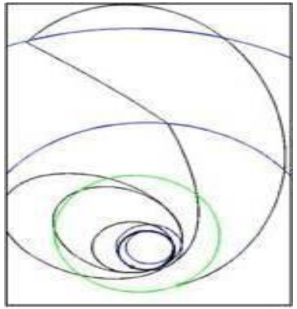
| | Team name | Value |
|---|---|---|
| | | |
| 1. | **Jet Propulsion Laboratory** | 1,850,000 |
| 2. | Deimos Space | 1,820,000 |
| 3. | GMV | 1,455,000 |
| | Moscow Aviation Institute | 1,364,000 |
| | Politecnico di Torino | 1,290,000 |
| | CNES/CS | 1,194,000 |
| | Glasgow University | 385,000 |
| | Moscow University | 351,152 |
| | Alcatel | 330,385 |
| | DLR | 330,000 |
| | Tsinghua University | 89,000 |

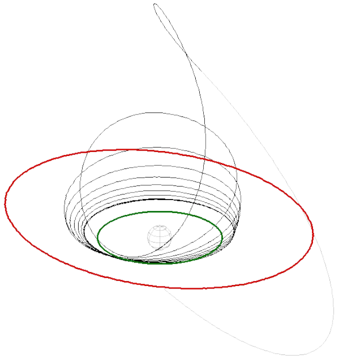
# GTOC2: Multiple Asteroid Rendezvous

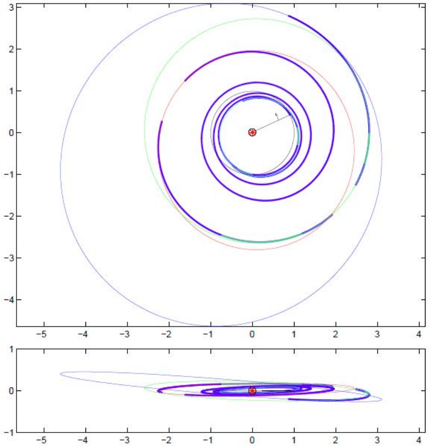


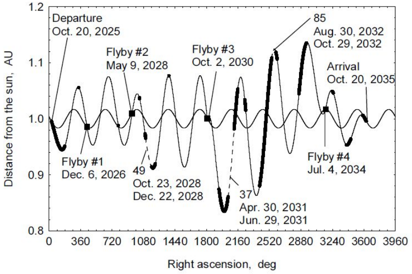
| | Team name | Value |
|---|---|---|
| | | |
| 1. | **Politecnico di Torino** | **98.64** |
| 2. | Moscow Aviation Institute and Khrunichev State Research | 87.93 |
| 3. | *Advanced Concepts Team (ESA)* | *87.05* |
| | Centre National d’Etudes Spatiales (CNES) | 85.43 |
| | GMV Aerospace and Defence | 85.28 |
| | German Aerospace Center (DLR) | 84.48 |
| | Politecnico di Milano | 82.48 |
| | Alcatel Alenia Space | 76.37 |
| | Moscow State University | 75.08 |
| | Tsinghua University | 56.87 |
| | Carnegie Mellon University | 27.94 |

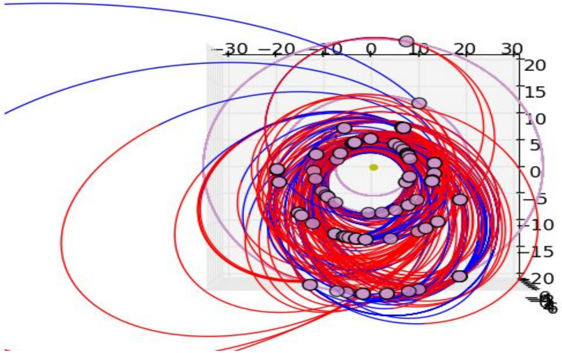
# GTOC: remarkable trajectories



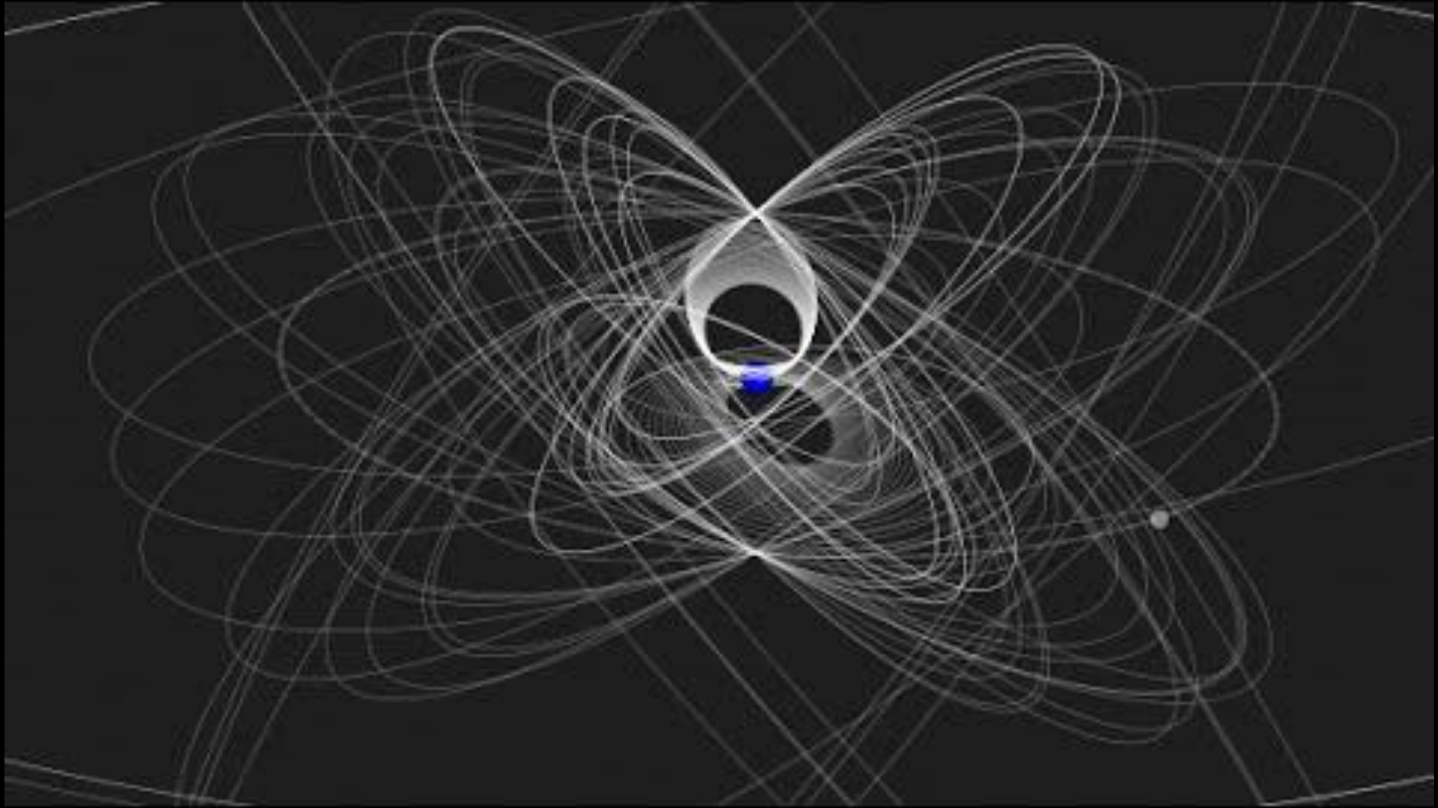




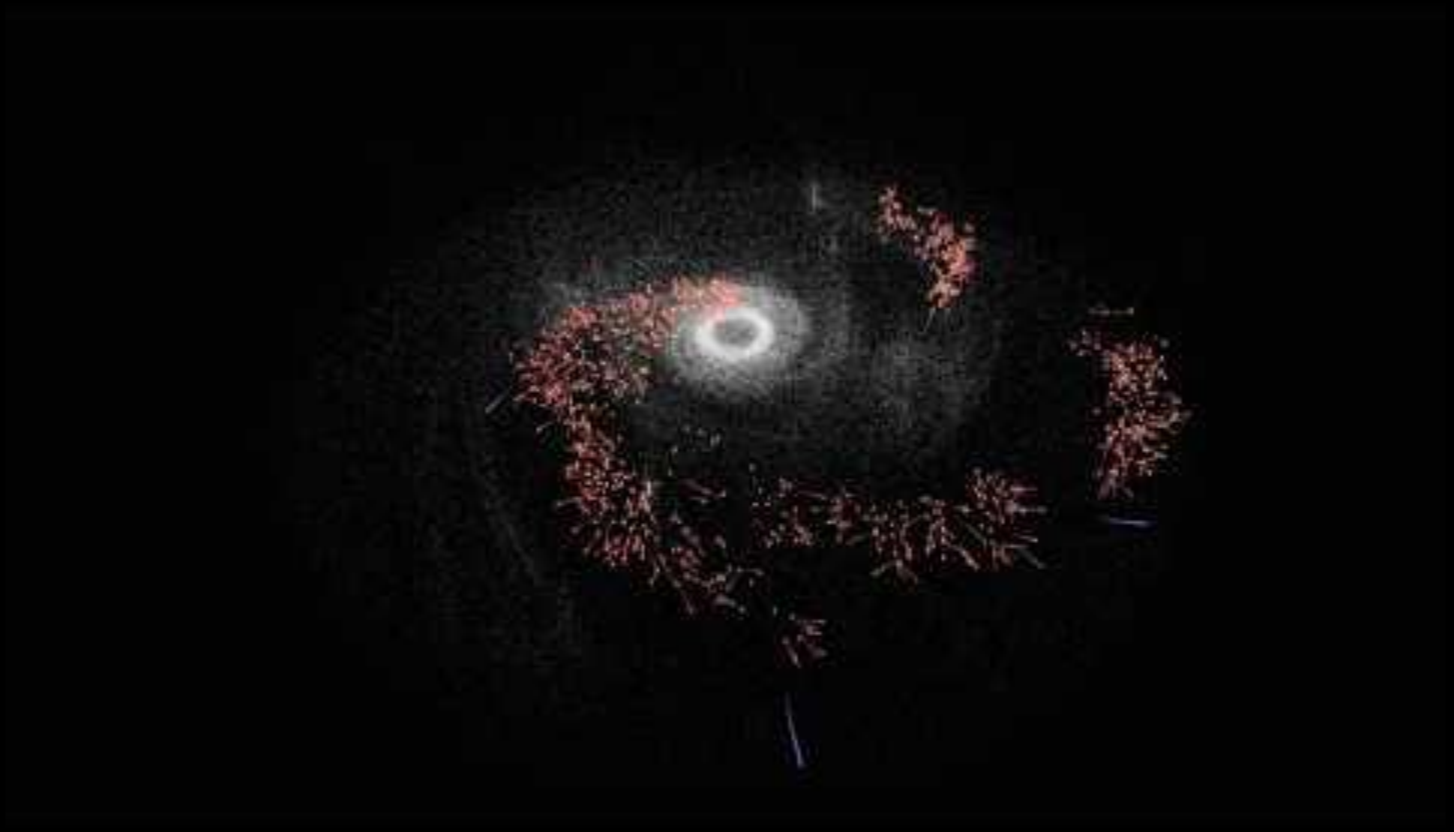


# GTOC 8



# GTOC X

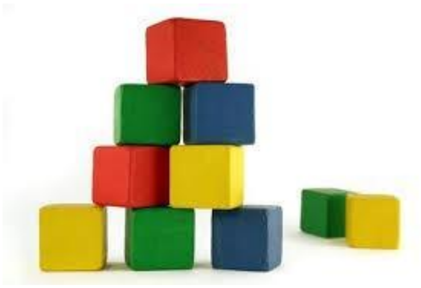


## It can (likely) all be evolved!

Some ESA (ACT) resources:

1. GTOC portal (contains the description of very difficult trajectory problems)
2. optimize.esa.int (a gym from ESA containing problems suitable for evolutionary approaches)
3. pykep (a C++/python module with trajectory problems building blocks)
4. pygmo2 (a C++/python module for massively parallel evolutions)
5. dcgpy (a python module for differentiable genetic programming)
6. This very tutorial!!

# … a representation problem, rocket science to the rescue!



# Building blocks

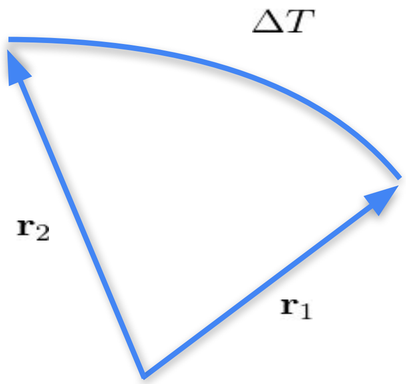
## ii. Lambert's Problem

```
import pykep
import numpy as np
l = pk.lambert_problem(r1 = [1,0,0], r2 = [0,1,0], tof = np.pi/2, mu = 1., cw = False, max_revs = 0)
```

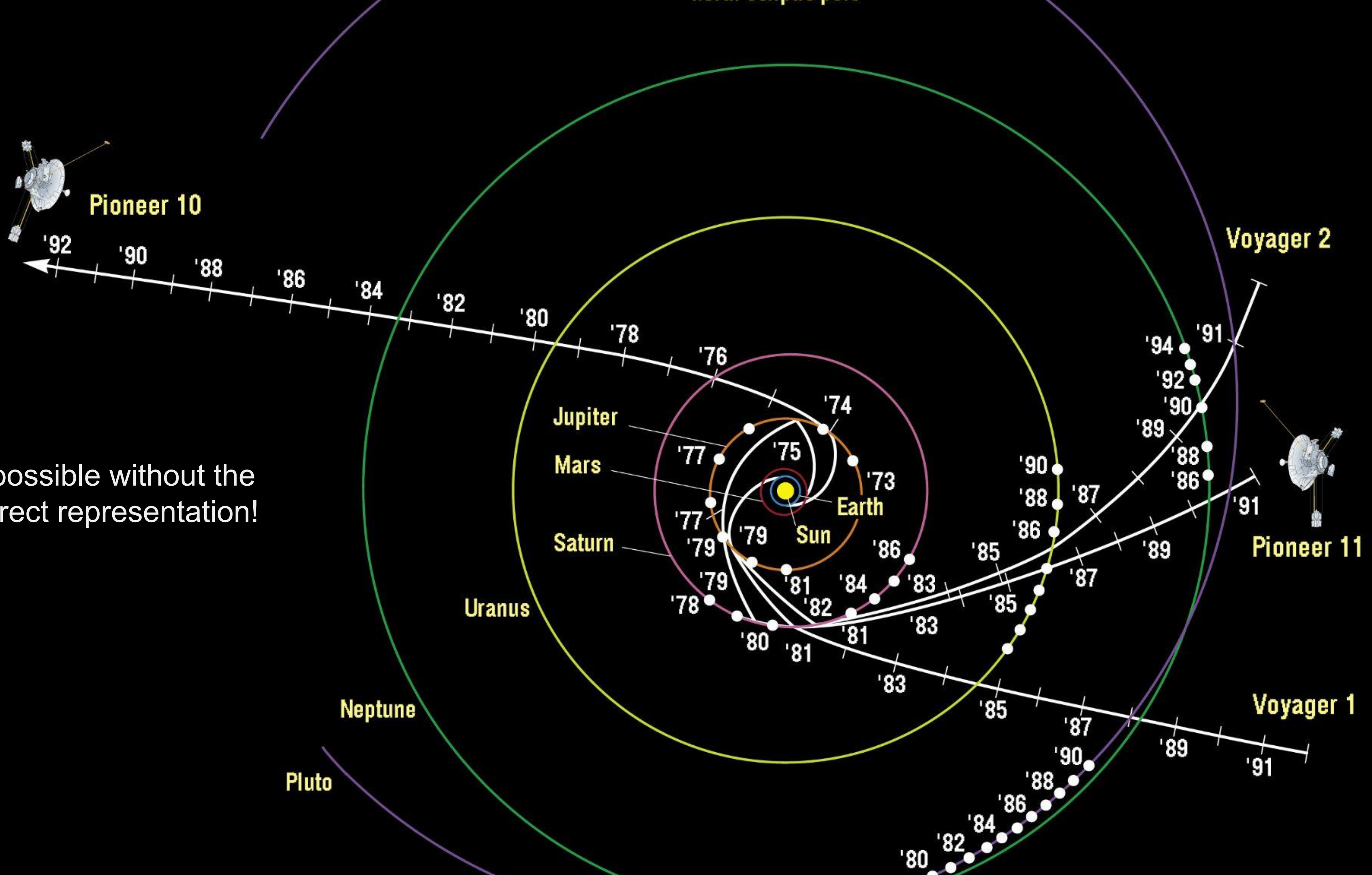
6.03 μs ± 169 ns per loop (mean ± std. dev. of 7 runs, 100000 loops each)

# Lambert's Problem

1. Going from one point to another in a fixed time.
2. It is, again, a TPBVP.
3. Its modern solution relies on results from Lambert, Gauss, Lagrange and in more modern times Battin, Lancaster and Blanchard
4. It turns out that all Lambert problems have 1 solution and, according to the transfer time, may also have 2·N multiple revolution solutions.

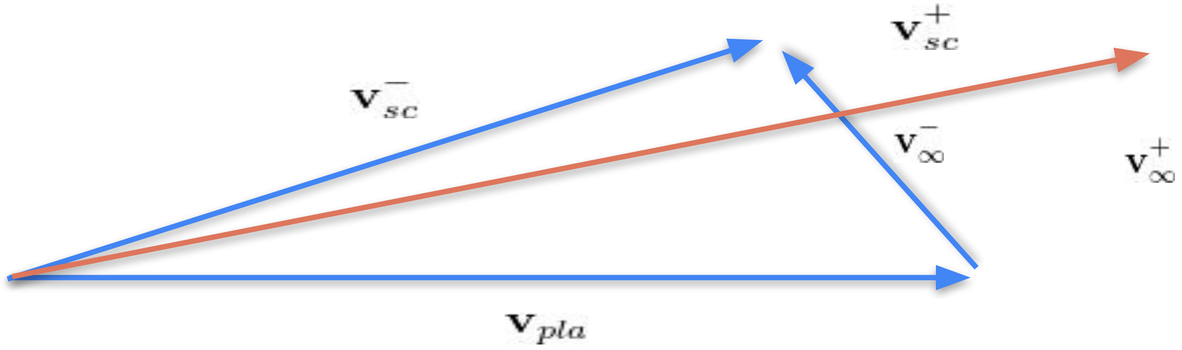



$$\begin{cases} \dot{\mathbf{r}} = \mathbf{v} \\ \dot{\mathbf{v}} = -\frac{\mu}{r^3}\mathbf{r} \\ \mathbf{r}(0) = \mathbf{r}_1 \\ \mathbf{r}(T) = \mathbf{r}_2 \end{cases}$$

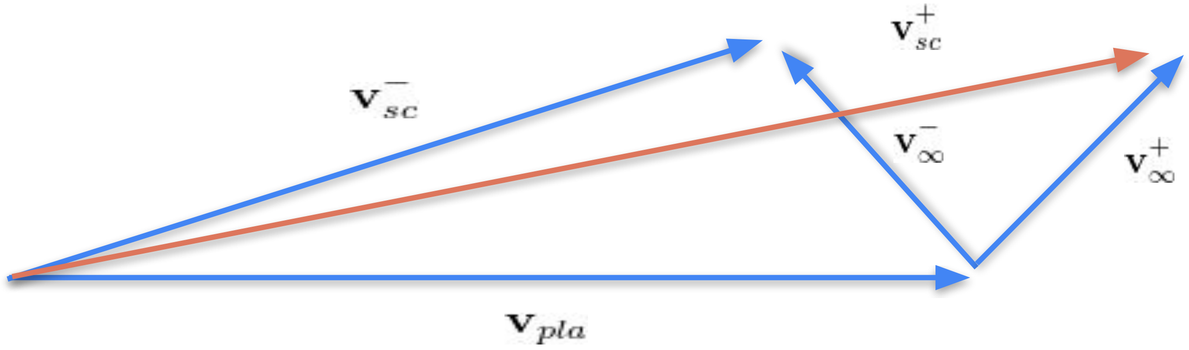



Impossible without the correct representation!

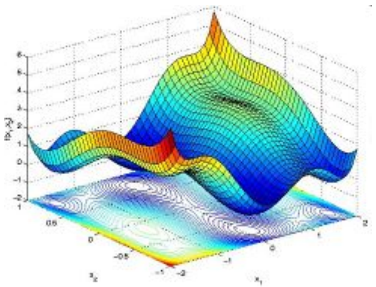
# Mivovitch sling-shot



# Mivovitch sling-shot

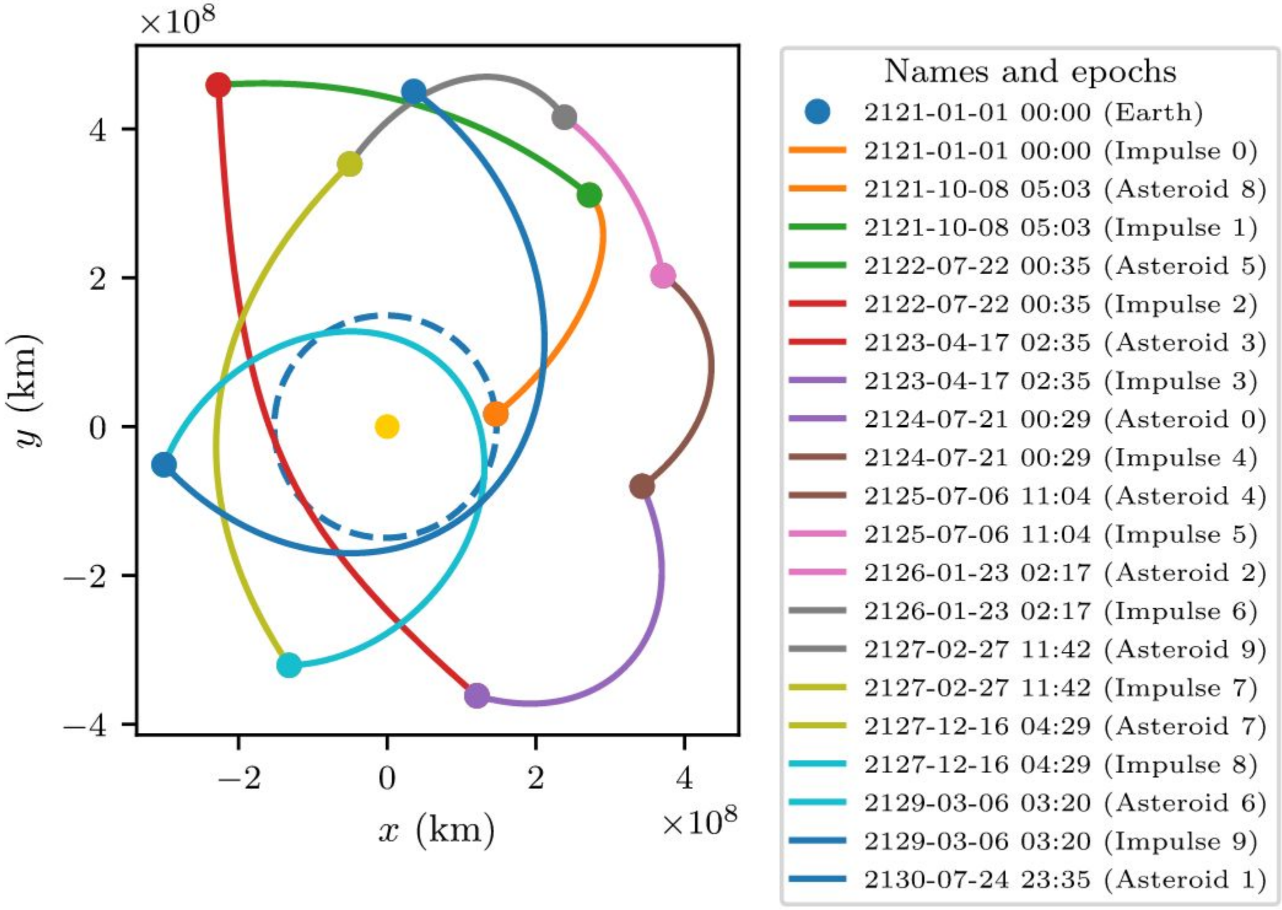



**Spacecraft velocity has changed in the absolute frame**



## Representations (encodings) developed so far …
## … in development

# Part II: Solution approaches to optimization problems in Space




from López-Ibáñez et al. (2022)

# Solution approaches to optimization problems in Space

- Exact (optimal) solutions often impossible / impractical

- ✅ Both gradient-based and gradient-free NLP solvers

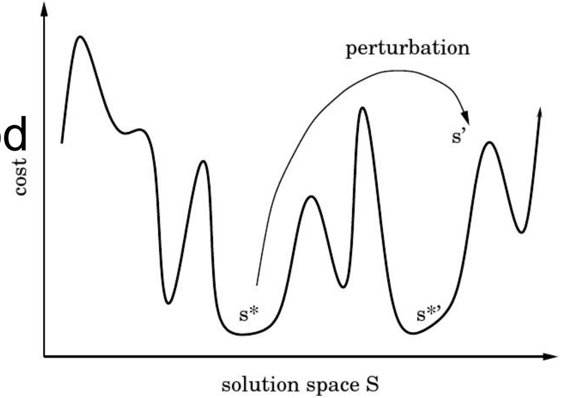
- ✅ Meta-heuristics:
  - Monotonic Basin Hopping (MBH),
  - jDE, CMA-ES, MOEA/D,
  - PSO, ACO
  - Tree search, e.g., Beam Search

- ✅ Hybrid methods: Beam P-ACO

# Non Linear Blackbox Numerical Search

- Continuous problems (MGA-LT):
  - Constrained ⇒ Penalised objective function
- Combinatorial + Continuous (interplanetary tours and multiple visits)
  - Given a fixed combinatorial solution (outer problem) optimize the continuous variables (inner problem)

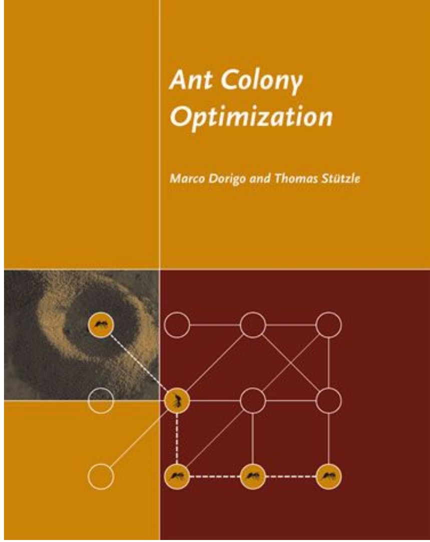
- NLP solution approaches (Yam et al., 2011):
  - Local search: fast deterministic NLP methods, e.g., SQP
  - Global search:
    - ➢ Simulated Annealing with adaptive neighborhood
    - ➢ Monotonic Basin Hopping (MBH) (≈ Iterated Local Search)



# Evolutionary Algorithms

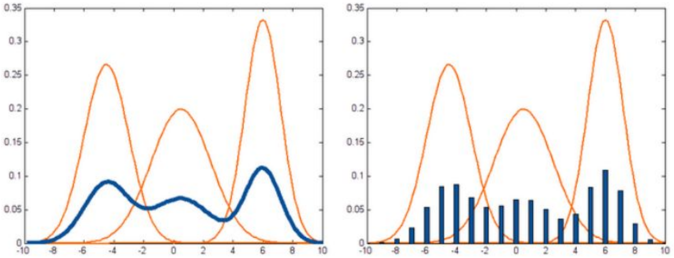
- GAs, DE and CMA-ES ⇒ by far the most popular approaches for solving trajectory optimization problems
- Hybridization and problem-specific operators:
    - Variable-length chromosomes (Gad, 2011)
    - Dynamic-size multi-population (Abdelkhalik & Gad, 2012)
    - Order-based GA for partial permutations (Izzo et al., 2014)
      ⇒ *"Hidden" genes*: chromosome contains a complete permutation
      but fitness is computed from a partial one
    - *Inver-Over Operator* (Tao & Michalewicz, 1998)
      modified for TSPs that are not invariant to cycling,
      e.g., debris removal (Izzo et al., 2015)
    - Self-adaptive jDE + various constraint handling techniques for MDA-1DM
      (Labroquère et al., 2014)

# Ant Colony Optimization



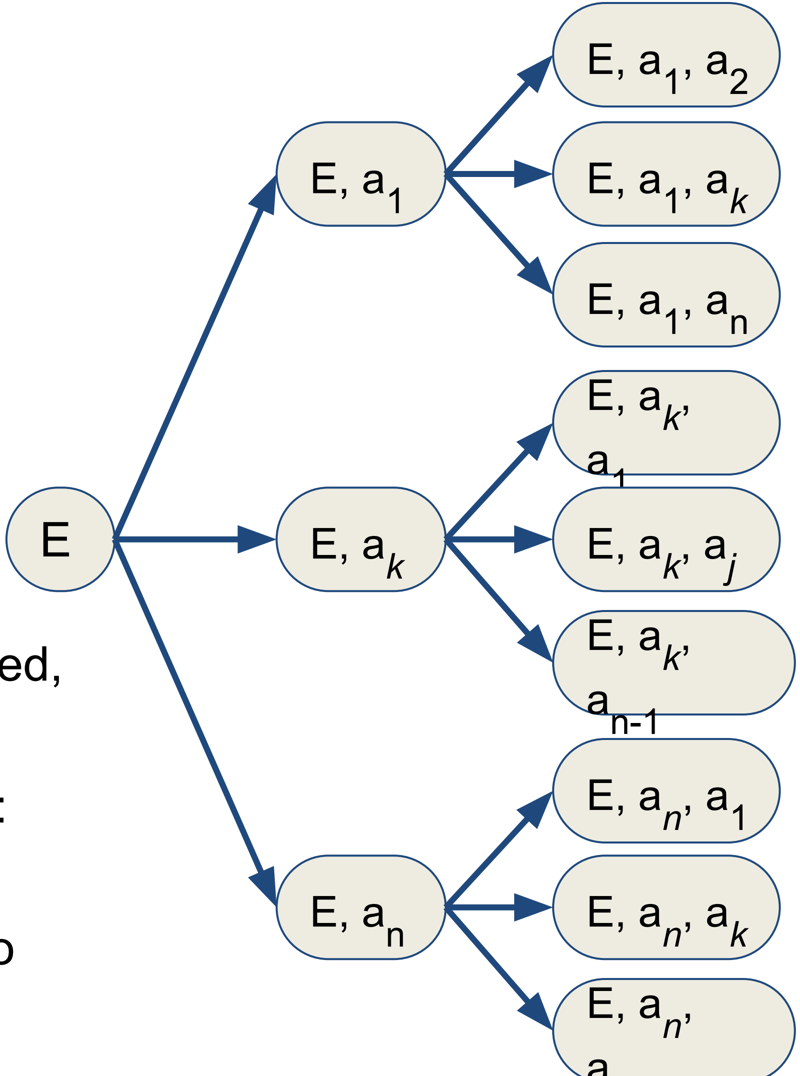

- Combinatorial formulations (select pairs of body & transfer):
  - Non-standard ACO for MGA (Ceriotti & Vasile, 2010)

- MIDACO: Mixed-Integer Distributed Ant Colony Optimization (Schlüter et al., 2013)
  - Black-Box Constrained Non-Convex Mixed-Integer
  - Based on *Extended ACO algorithm*:
    Samples solutions from multi-kernel Gauss PDFs
    (similar to ACOR by Socha & Dorigo, 2008)
  - Coupled with the *oracle penalty method*
    for constraint handling
  - Hybridized with local deterministic SQP
    for further optimizing continuous variables



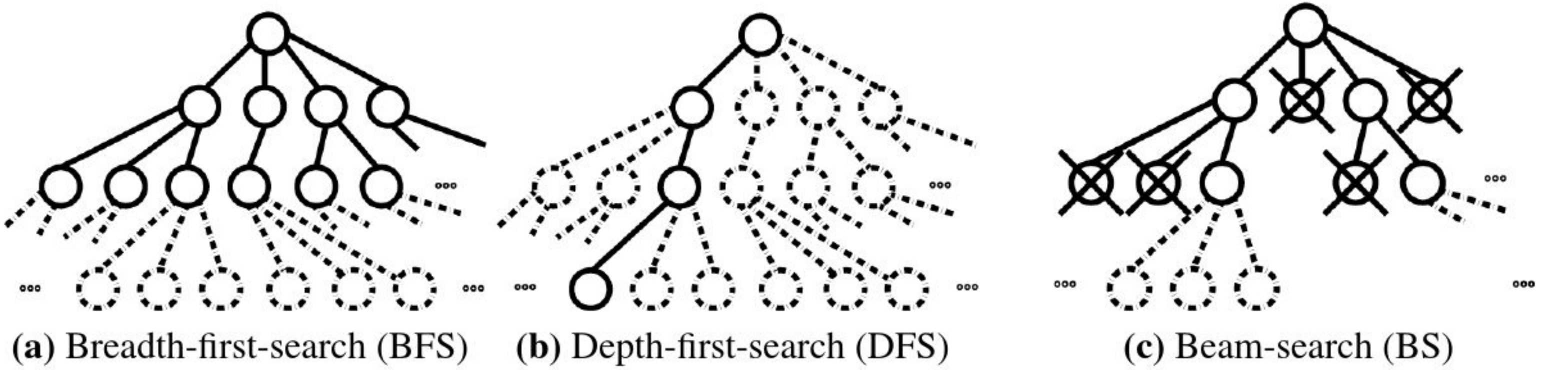
# Tree Search Methods

- Combinatorial: Tours and multiple visits
- Each node is a (partial) trajectory
- Construct solution incrementally adding a trajectory leg (fly-by, rendezvous, …)
- Branching typically involves solving a numerical inner problem to execute the maneuver, e.g., Lambert's problem.
- The inner problem may need to be optimized, e.g., find optimal *tof* and *ΔV* using NLP
- We CANNOT exhaustively search the tree:
  complete DFS or BFS are *impractical*
- Probabilistically/heuristically decide what to branch and what branches to prune



# Tree Search Methods

from Izzo et al (2016)

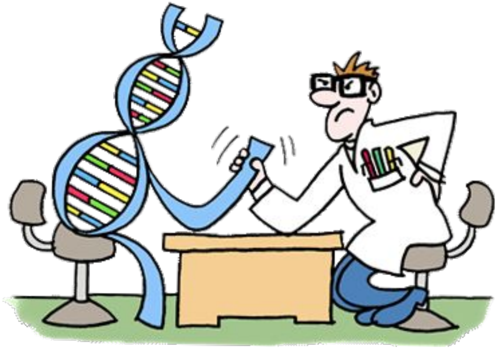



- DFS + branch pruning criteria (Izzo et al., 2014):
  - remaining fuel/time;
  - partial mission score;
  - best complete solution found so far, …

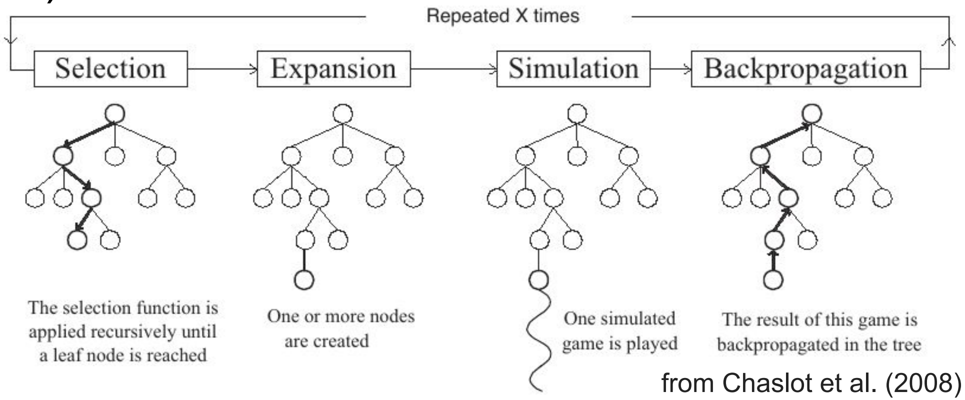
- Beam-search: rank nodes at equal depth and prioritize what to expand
  - Winner of GTOC5 (Petropoulos et al., 2014)
  - MO version applied to GTOC7 (Izzo et al., 2016)
  - Beam-size × Solving time of inner problem ≈ Time exploring next depth
  - In some problems partial trajectories are *not comparable* (e.g, GTOC6)

# Other Tree Search Methods

- ✘ DFS + pruning criteria: Difficult to estimate running time
- ✘ Beam-search: Too greedy if nodes at same depth are not comparable



- Lazy Race Tree Search (Izzo et al., 2013):
  - Beam composed of nodes of different depth but within same mission time window
  - GTOC6, Gold “Humies” Award at GECCO 2013
- Monte Carlo Tree Search (Hennes & Izzo, 2015)
  - Heuristic-free selection policy (UCB)
  - Expansion: add random node
  - Simulation: stochastic sampling
    - *add all nodes to the tree*
  - Backprop: update policy
  - *Contraction: prune completed subtrees*



from Chaslot et al. (2008)

# Hybrid methods

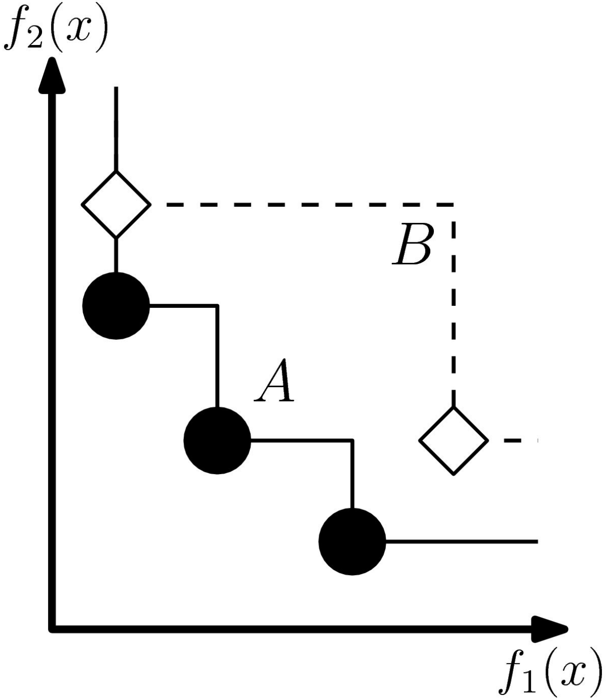
# Hybrid methods

- Combinatorial + Continuous Hybrids

- Tree search + ACO:
  - ➢ **Beam P-ACO**: Beam Population-based-ACO (Simões et al., 2017) inspired by Beam-ACO from classical constrained sequence-based problems (Blum, 2005; López-Ibáñez & Blum, 2010)

- Different metaheuristics at the outer and inner level:
  - ➢ **Integer GA** (outer: optimal flyby sequence) + Cooperative PSO-DE (inner: transfer) for MGA (Englander et al., 2012)

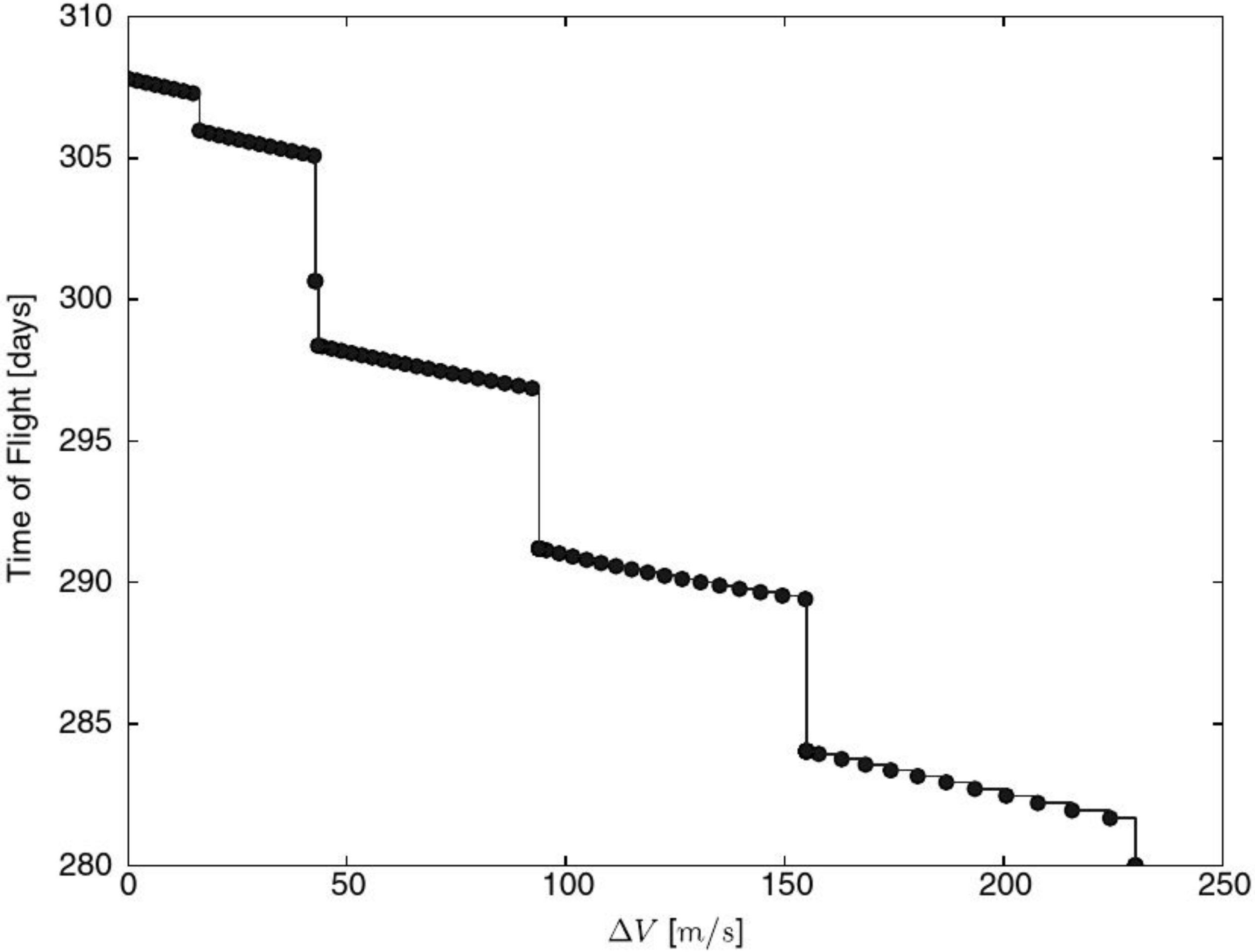
# Multi-objective approaches




- Many problems (and inner problems) have multiple conflicting objectives:
  - Total cumulative velocity increment $\Delta V_{tot}$
  - Total time of flight (*TOF*)
  - Number of bodies visited
- Typically aggregated into a single "mission score"
- Few multi-objective approaches:
  - MOEA/D + seeding with extreme points: min $\Delta V$ and min *TOF* (Izzo et al., 2015b)
  - MHACO = MIDACO + Nondominated sorting + HV contribution (Acciarini et al. 2020)
    - ➢ Outperforms MOEA/D and NSGA-II on 4 trajectory optimization problems

# Multi-objective approaches

(Izzo et al., 2015b)



# Space-specific heuristics

- Preprocessing steps that reduce search space size
  e.g., reduced box-bounds by gravity assist space pruning for MGA (Izzo et al., 2007)
- Nearest neighbor distances between orbital bodies:

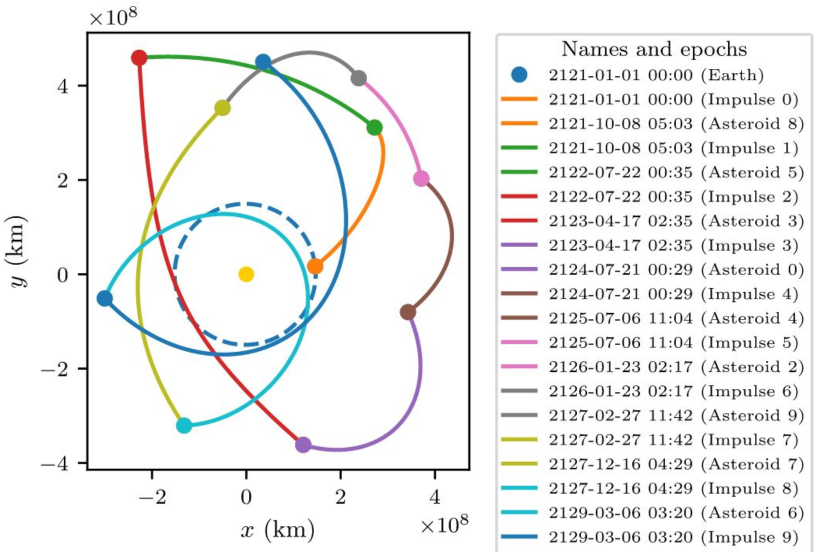
✘ *Euclidean distance* ignores relative velocity

✔ *Orbital distance* (phasing) indicators (Izzo et al., 2016):

Estimates “distance” at departure time

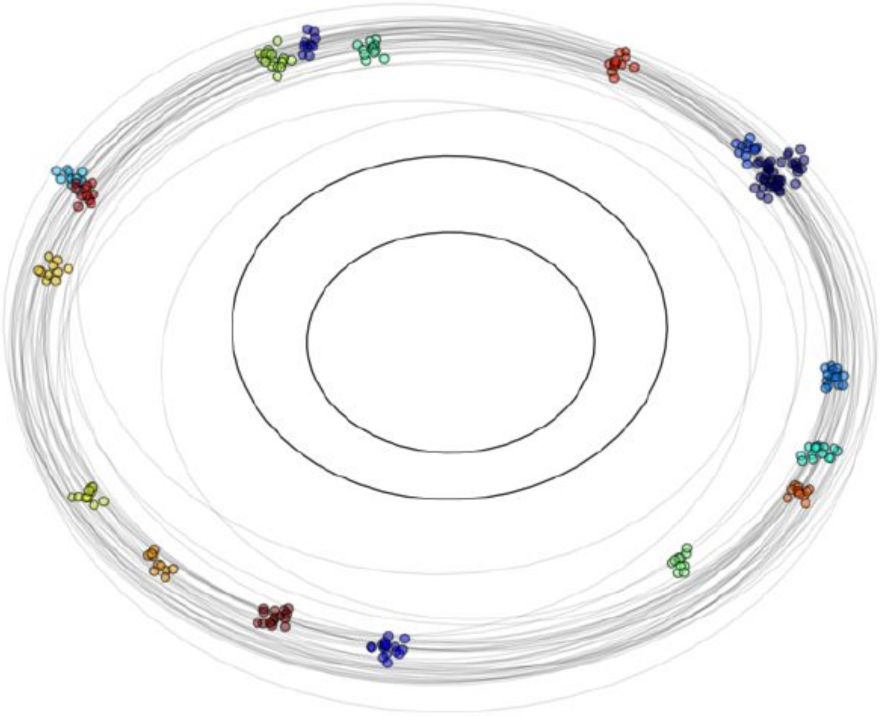
👍 *Improved orbital distance* (Simões et al., 2017):

Also takes into account the “distance” at arrival time



# Space-specific heuristics

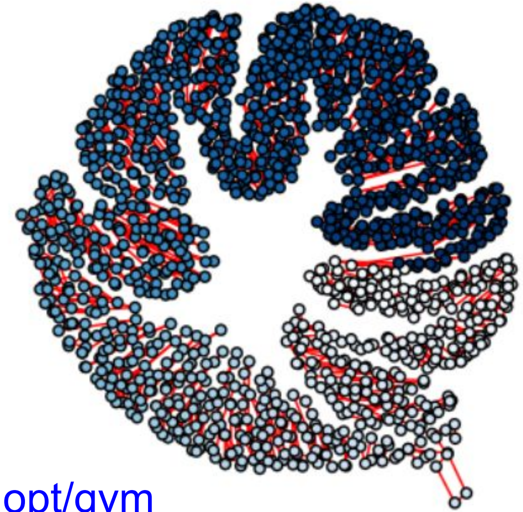
- Clustering (e.g., using the above indicators)
  - ➢ DBSCAN (Izzo et al., 2016)
- Cluster pruning (Izzo, 2010)
  - ➢ Define/update box/distance bounds to focus on promising areas



# Solution approaches to optimization problems in Space

- MGA
  - ➢ jDE, CMA-ES, PSO
- MGA-1DSM
  - ➢ GA: NASA Versatile ImpulSive Interplanetary Trajectory OptimizeR (VISITOR)
  - ➢ CMA-ES with smart restarts: https://github.com/dietmarwo/fast-cma-es
- Multi-objective variants
  - ➢ MHACO or MOEA/D or NSGA-II + seeding with single-objective extremes
- MGA-LT
  - ➢ MBH (Monotonic Basin Hopping), EAs + numerical local search (memetic)
- Asteroid tours and TSP variants
  - ➢ beam search, beam P-ACO, lazy race tree search,
    order based GAs, inver-over operator, etc....

# Existing benchmarks


(Izzo et al., 2015)

- Optimize ESA Platform: https://optimize.esa.int
  - Miscellanea of problems, leaderboard based.
- GTOC: Global Trajectory Optimization Competition
  - Complex and large interplanetary trajectory problems
  - 12 editions so far (https://gtoc12.tsinghua.edu.cn)
- pykep gym: https://github.com/esa/pykep/tree/master/pykep/trajopt/gym
  - Miscellanea of problems.
- Tours and TSP variants
  - TSP, TSP-CS, TSP-DCS for: Active space debris removal trajectory design
  - Multi-rendezvous Spacecraft Trajectory Optimization
  - Asteroid Routing Problem: https://github.com/MLopez-Ibanez/AsteroidRoutingProblem
  - SpOC: Mining: Mine the Belt
- Global Trajectory Optimisation Problems Database (GTOP, no longer maintained):
  - Miscellanea of problems (https://www.esa.int/gsp/ACT/projects/gtop/)

# Conclusions

✅ Lots of interesting optimization problems with “unusual” features

✅ Lots of benchmarks, simulation tools and software available

✅ Competitions and challenges

# Conclusions

- ✅ Lots of interesting optimization problems with “unusual” features
- ✅ Lots of benchmarks, simulation tools and software available
- ✅ Competitions and challenges

# Optimization Challenges at the European Space Agency

**Dario Izzo**
**European Space Agency**
Dario.Izzo@esa.int

**Manuel López-Ibáñez**
**University of Manchester, UK**
manuel.lopez-ibanez@manchester.ac.uk
http://lopez-ibanez.eu

The University of Manchester
Alliance Manchester Business School

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Vasile, Massimiliano, and Paolo De Pascale. "Preliminary design of multiple gravity-assist trajectories." *Journal of Spacecraft and Rockets* 43, no. 4 (2006): 794-805.

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- See the collection in the portal: https://sophia.estec.esa.int/gtoc_portal/?page_id=312

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# Expensive Black-Box Permutation Optimization

**Black-box**: $f(\pi)$ = 

**Expensive**:

- Each evaluation is costly !
- Between 100 – 1000 evaluations

**Permutation-based**

- $\pi \in S_n$
- e.g., $\pi = (3, 2, 4, 5, 1)$
- Either representing an order or a ranking.

- Previous works
  - Surrogate-based (CEGO) [Zaefferer et al., 2014]
  - Ant Colony Optimization and ACO+surrogate-model: [P´erez C´aceres et al., 2015]
  - Unbalaced Mallows Model (UMM): [Irurozki & L´opez-Ib´an˜ez, 2021]
- Typically benchmarked on: TSP, QAP, LOP, PFSP . . .

*Real-world* expensive black-box permutation problems ?